# QMath-8. Mathematical Results in Quantum Mechanics

## Abstracts

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# Featured Topics:

Bound state problems and scattering theory for Schrödinger operators. Inverse spectral and scattering theory of Schrödinger operators. Nonlinear Schrödinger equations. Quantum chaos, quantum dots and wave guides. Parameter dependent Hamiltonians. Spectral and localization properties of Schödinger operators

# Contents

## 1 R. Adami: Blow-Up solutions for schroedinger equation with concentrated nonlinearity

In this talk we discuss some results on a class of one and three dimensional nonlinear Schrödinger equations. The nonlinear term consists of a finite sum of Dirac deltas whose coupling constants are functions of the solution. We show how a blow-up phenomenon can occurr and describe some different families of blow-up solutions. Finally, we give an estimate on the blow-up rate and investigate the links between this problem and the corresponding one for the standard nonlinear Schroedinger equation.

## 2 M.A. Antonets: Negative discrete spectrum of Dirac operator for contact interaction with plane

The negative discrete spectrum of the Dirac operator is considered in the case of an electron interacting by contact way with a plane in the presence of tilted magnetic field. For resolvent of this operator corresponding Krein formula is established. Then it is used to prove existence of the negative discrete eigen-values and constructing of their asymptotics for large constant of the contact interaction. The work is based on some new results of the theory of pseudodifferential operators [1].

**References.**

1. Antonets M.A. Initial-value problem for pseudodifferential operators. Journal of Mathematical Science,Vol. 98, No. 6,March 2000, p 629-653.

## 3 B.Apagyi: Potentials from phase shifts at fixed energies by a non-linear inversion method

With Z. Harman and W. Scheid. We apply the Cox-Thompson (CT) quantum inversion method at fixed energy to derive potentials from given sets of phase shifts. The resulted non-linear equations are solved by the root-finder package of the Mathematica, the inversion potentials are then constructed with the aid of the Maple. The test examples involving box and Woods-Saxon potentials show the superiority of the CT method over the well known Newton-Sabatier inversion procedure which assumes linear equations for the solution.

## 4 N. Atakishyev: Mellin transforms for some families of q-polynomials

We discuss q-analogues of Euler's reflection formula and gamma integral. Ramanujan's q-extension of the Euler integral representation for the gamma function plays a central role in this work and it enables us to derive the Mellin integral transforms for some families of basic hypergeometric orthogonal polynomials from the Askey scheme.

## 5 B. Bodmann: Transformation formula relating resolvents of Berezin-Toeplitz operators by an invariance property of Brownian motion

Using a stochastic representation provided by Wiener-regularized path integrals for the semigroups generated by certain Berezin-Toeplitz operators, a transformation formula for their resolvents is derived. The key property used in the transformation of the stochastic representation is that, up to a time change, Brownian motion is invariant under conformal mappings.

## 6 U. Carlsson: Schroedinger operators with several potential wells in the semi-classical limit

Consider first a Schroedinger operator with exactly one point-well. Then, if the Hessian is positive definite at the point, it is well-known that the lowest eigenvalue is simple, and that the corresponding eigenfunction is positive up to a non-zero constant factor. In the case of a double-well potential the eigenvalue splits. When analyzing the spectrum of an operator with several wells we use one-well operators as reference operators. In the case of infinitely many wells conditions on the potential are needed which in turn imply geometric restrictions on the distribution of the wells. Of special interest to us is the case when the wells have a hierarchical configuration. Throughout the talk I restrict my study to the structure of the bottom of the spectrum. One question of interest to me is if the double-well splitting phenomenon also appear repeatedly in the hierarchical case.

## 7 M. Castagnino: The Classical limit

The quantum to classical–statistical limit is studied. It is proved that the limit $\bar{h} \to 0$ coincides with this limit for the operators algebra but it is not so for the states convex set. In the later case, to obtain the classical limit, decoherence must be invoked. It is well known that, once upon a time, the quantum to classical limit was considered just the limit $\bar{h} \to 0$ (as the relativistic to classical limit is just $c \to \infty$). Bohr and Heisenberg stressed this analogy but Einstein was unmoved by such arguments which considered an oversimplification, since the statistical structure of quantum mechanics is completely different to the individual particle one of classical mechanics ( actually, Einstein was right, the experience showed that matters were much more difficult, since in the former limit many physical phenomena must be considered (decoherence, localization, correlations, etc.) making the subject quite complicated. See e. g. [1]. Latter a "classical statistical" structure for quantum mechanics was found by Wigner [2] (i. e. quantum mechanics formulated on phase space) and it was shown that it is convenient to decompose this limit into the quantum to classical statistical limit and the classical statistical to classical limit (see [3]). In this talk, using the methods of papers [4], [5] and [6], we will consider the quantum to classical statistical limit for a system with continuous evolution spectrum. The idea is to show, in the simplest but complete way, the essence of the limit, trying to clear the matter as much as possible.

**References.**


1. L. E. Ballentine, *Quantum mechanics*, Prentice Hall, Englewoods Cliffs, 1990.
2. M. Hillery, R.F. O'Connell, M. O. Scully, E.P. Wigner, Phys. Rep., **106**, 123, 1984.
3. M. Castagnino, R. Laura, Phys. Rev. A, **62**, 022107, 2000.
4. R. Laura, M. Castagnino, Phys. Rev. A, **57**, 4140, 1998.
5. R. Laura, M. Castagnino. Phys. Rev. E, **57**, 3948, 1998.
6. M. Castagnino, M. Gadella, R. Laura, R. Id Betan, Phys. Lett. A, **282**, 245, 2001.


## 8 M. Combescure: Semiclassical results in the linear response theory

With Didier Robert. We consider a quantum system of non-interacting fermions at temperature T, in the framework of linear response theory. We show that semiclassical theory is an appropriate framework for describing some of their thermodynamic properties, in particular through exact expansions in Planck constant h of their thermodynamic susceptibilities. We show how the orbits of the classical motion in phase space manifest themselves in these expansions, in the regime where T is of order of h.

## 9 M. Demuth: Capacities and equilibrium potentials in scattering theory

Regular Dirichlet forms are associated to selfadjoint operators and to Hunt processes. The one-equilibrium potential realizes the infinium in the definition of the capacity. The equilibrium potential has a stochastic representation using the first hitting time of an obstacle. In obstacle scattering Dynkin's formula describes resolvent differences in terms of the harmonic extension operator. This can be used to study Hilbert-Schmidt or trace class properties of such resolvent differences. In scattering theory the scattering matrix gives the main link to physical applications. In case of perturbations by two different obstacles the difference of the scattering matrices is estimated by the capacity of the symmetric difference of the obstacles. The same follows for the scattering phases. This kind of quantitative estimates can be extended to large coupling problems where the decrease of the scattering phases depends on the Laplace transform of the occupation time.

## 10 J. Dittrich: Quantum waveguides with combined boundary conditions

The existence of bound states is studied for two examples of straight quantum waveguides with Dirichlet and Neumann boundary conditions imposed on different parts of the boundary. Cases with and without bound states are found.

## 11 V. Enss: Quantum scattering in a rotating potential

We study scattering theory for a quantum particle under the influence of a classical rotating body. The interaction is described by an explicitly time-dependent potential obtained by a uniform rotation of a fixed potential. Under mild regularity and suitable – possibly unisotropic – decay assumptions we investigate existence and completeness of wave operators and, in particular, the energy transfer between the particle and the rotating body. We show that the kinetic energy is uniformly bounded on scattering states and we determine the loss or gain of energy for special classes of states.

Joint work with Vadim Kostrykin and Robert Schrader.

## 12 P. B. Espinoza: Moyal-like evolution equations for SU(2) quasi-distribution functions.

We derive a differential form of the Star Product for a s-parametrised family of quasi-distributions of SU(2). We apply these results to find evolution equations for dynamical systems described by polynomial Hamiltonians of SU(2) algebra generators.

## 13 P. Exner: A norm-resolvent approximation of strongly singular interactions

We discuss a family Schrödinger operators with potentials scaled on a specific nonlinear way which approximates the $\delta'$-interaction Hamiltonian in the norm-resolvent sense. This approximation, based on a formal scheme proposed by Cheon and Shigehara, has nontrivial convergence properties which are in several respects opposite to those of the Klauder phenomenon.

## 14 S. Fournais: Regularity of molecular eigenfunctions and densities

This is joint work with Maria Hoffmann-Ostenhof, Thomas Hoffmann-Ostenhof and Thomas Østergaard Sørensen. We study the regularity of molecular eigenfunctions near singularities of the potential. Using a Jastrow-type Ansatz, we can describe precisely the singularities of the eigenfunction up to class $C^{1,1}$. As an application we prove that the electron densities of molecular eigenfunctions are smooth away from the nuclei.

## 15 S. Fulling: The quantum theory of ceilings and floors

The semiclassical analysis of the propagator of a particle subject to both a linear potential and a reflecting boundary is surprisingly nontrivial, and it provides a prototype for the study of systems with both potentials and boundaries, as the Airy problem does for ordinary turning-point problems.

## 16 E. Giere: Asymptotic completeness for functions of the Laplacian perturbed by potentials or obstacles

Using scattering theoretical methods we show that the absence of the singular continuous spectrum of perturbations of functions of the Laplacian. In an application we study the fractional Laplacian $(-\Delta)^{\alpha/2}$ and its perturbation by short range potentials or obstacles. For the latter we give an example with an unbounded set as an obstacle.

## 17 C. Hainzl: General decomposition of radial functions on $\mathbf{R}^n$ and refined conditions for positive definiteness

We present a generalization of the Fefferman-de la Llave decomposition of the Coulomb potential to quite arbitrary radial functions $V$ on $\mathbf{R}^n$ going to zero at infinity. As a byproduct, we obtain conditions for positive definiteness of $V$, thereby improving results of Askey. In a subsequent paper we use this to describe assymptotic exactness of Hartree-Fock approximation of Yukawa systems in the high density limit.

## 18 B. Hall: Coherent states on spheres

I will describe joint work with Jeffrey Mitchell on coherent states for a quantum particle whose classical configuration space is a sphere. These coherent states are not of Perelomov type.

## 19 I. Herbst: Quantum scattering with potentials independent of $|x|$

Scattering theory is developed for an n dimensional particle under the influence of a potential which is homogeneous of degree zero for large x and such that V restricted to a large sphere is a Morse function. We describe the somewhat unusual asymptotic behavior of the wave function as it develops in time and define appropriate asyptotic dynamics in certain energy regimes. We prove asymptotic completeness of the corresponding wave operators. We describe similar but geometrically more complex behavior of quantum systems in magnetic fields which are homogeneous of degree -1. This work is joint with Erik Skibsted and partially with Horia Cornean.

## 20 M. Hitrik: Eigenfrecuencies and expansions for dissipative wave equations

We study eigenfrequencies and propagator expansions for damped wave equations on compact manifolds. In the strongly damped case, the propagator is shown to admit an expansion in terms of the finitely many eigenmodes near the real axis, with an error exponentially decaying in time. In the presence of an elliptic closed geodesic not meeting the support of the damping coefficient, we show that there exists a sequence of eigenfrequencies converging rapidly to the real axis. In the case of Zoll manifolds, the set of all eigenfrequencies is shown to exhibit a cluster structure determined by the Morse index of the closed geodesics and the damping coefficient averaged along the geodesic flow. We then show that the propagator can be expanded in terms of the clusters of eigenfrequencies in the entire spectral band.

## 21 M. Horváth: A necessary and sufficient condition in inverse eigenvalue problems

Consider the Schrödinger operator $Ly = -y'' + q(x)y$ on $x \in [0, \pi]$. We investigate the problem of reconstructing the potential $q(x)$ by a sequence of eigenvalues $\lambda_n$, $Ly_n = \lambda_n y_n$, where the eigenfuctions satisfy a fixed right boundary condition at $\pi$ and possibly different arbitrary left boundary conditions. Among others we have the following

<u>*Theorem*</u> *Suppose that* $y_n(\pi) = 0$ *and that the potential is known on the subsegment* $(0, a)$ *with some* $0 \leq a < \pi$. *Let* $\mu \neq \pm\sqrt{\lambda_n}$. *The eigenvalues* $\lambda_n$, $n \geq 1$ *determine the potential* $q(x)$ *if and only if the system* $\{\exp(\pm 2i\mu x), \exp(\pm 2i\sqrt{\lambda_n}x) : n \geq 1\}$ *is complete in* $L_2(a - \pi, \pi - a)$.

By the aid of this statement most of the formerly known sufficient conditions can be converted into standard completeness results.

## 22 V. K. Ignatovich: Apocrypha of the standard scattering theory

The standard scattering theory contradicts the principles of canonical quantum mechanics. The scattering theory, which rigorously follows principles of quantum mechanics gives not cross sections, but dimensionless scattering probabilities. With dimensionless scattering probabilities it is impossible to describe transmisson of a sample. To describe the transmission we must introduce the area of wavefront of scattered particle, which leads us to a nonspreading wave packet. The best candidate for such a packet is the Broglie singular wave packet. Some physics for such a packet is presented. Also some results for scattering of neutrons in a monoatomic gas are presented that differ from predictions of standard theory.

## 23 V. Ivrri: Function in the box and logarithmic uncertainty principle

Microlocalization (and Microlocal Analysis in general) require logarithmic uncertainty principle: $\rho \times \gamma \geq Ch|\log h|$ where $\rho$, $\gamma$ are scales with respect to momenta and coordinates respectively, $h$ is Plank's constant.

## 24 E. Kaikina: Asymptotic behavior for Korteweg-de Vries-Burgers equation on

**half-line without smallness of initial data**

We study the initial-boundary value problem for the Korteweg-de Vries-Burgers (KdVB) equation without smallness condition on the data. The KdVB equation has many physical applications. There are many works on the large time asymptotics of solutions in the case of small initial data for the Cauchy problem. By virtue of the difficulty of the study of not small solutions of nonlinear problem globally in time there are few results on the asymptotic behavior of solutions with large initial data. Due to the zero boundary value we will obtain that solutions have more rapid time decay in comparison with the corresponding Cauchy problem. Therefore using the symmetry property of the nonlinearity we estimate the $L$ - norm of solutions without any assumption on the size of the initial data. Also we apply the energy method and estimates of Green function to prove smallness of the uniform norm of the solution after some time and prove global existence and asymptotic formulas for solutions.

## 25 A. Komech: On attractor of a nonlinear U(1)-invariant 1D Klein-Gordon equation

An attractor is studied for all finite energy solutions to a model nonlinear U(1)-invariant 1D Klein-Gordon equation. The attractor is a union of the solitary waves $\psi(x)\exp(i\omega t)$.

## 26 M. Kovalyov: Spectral decomposition of potential for the one-dimensional Schroedinger operator

If the spectrum of the 1-dim Schrodinger Operator (SO) consists of a finite number of points, the coresponding potential must be a nonliner superpositin of a finite number of solitons. Is a similar assertion valid for continuous spectrum, i.e. if the spectrum is continuous can the potential be represented as nonlinear superposition of some special soliton-like solutions? Can these solutions be used to construct a potential with required a priory given properties? These questions will be discussed and graphic illustrations will be shown.

## 27 D. Krejcirik: Bound states in curved quantum layers

We consider a quantum particle constrained to a tubular neighbourhood of constant width built over a curved non-compact surface embedded in $\mathrm{R}^3$. We suppose that the latter is asymptotically planar and that such a layer has the hard-wall boundary. Under these assumptions we find sufficient conditions which guarantee the existence of geometrically induced bound states. This is a joint work with Pierre Duclos and Pavel Exner.

## 28 I. Krasovski: Spectral estimates for discrete one-dimensional periodic Schrödinger operators

We obtain the estimates for the boundaries of the spectrum and the integral estimates for the measure of the gaps of periodic operators on $l^2(Z)$: $(H\psi)(n) = \psi(n+1) + \psi(n-1) + V_n\psi(n)$, where the real potential $V_n$ has a period $q$, that is $V_n = V_{n+q}$, for any $n \in Z$. We construct and study the quasimomentum $k(z)$ corresponding to this operator. Our results are obtained from the properties of $k(z)$ considered as a conformal mapping of complex domains.

## 29 V. V. Kucherenko: Spectral asymptotics for the $N$ particles Schrödinger equation with periodic binary potential

With Salvador Arellano Balderas. We consider the Boson's eigenfunctions for the $N$ particles Schrödinger spinless equation on the torus $M = (\mathbf{R}^3/\mathbf{Z}^3)^N$:

$$\sum_{j=1}^{N} -\epsilon^2 \Delta_{x_j}\psi + \sum_{1\le i<j\le N} v(|x_i - x_0|)\psi = E\psi \tag{1}$$

Let us $L_2^s(M)$ is the symmetrical subspace of $L_2(M)$; therefore $\psi \in L_2^s(M)$. We suppose that

$$v(|x-y|) = \sum_{|k|=0} \nu_k \exp\{i2\pi(k,(x-y))\} \tag{2}$$

$$\mathrm{Im}\nu_k = 0, \nu_k = \nu_{-k}, \nu_k \geq 0, \sum_{|k|=0} \nu_k |k|^4 < \infty.$$

The supposition $\nu_k \geq 0$ corresponds to the repulsive case.

It is well known that any symmetrical function $\psi(x_1,\dots,x_N) \in L_2^s(M)$ can be decomposed into the Fourier serie with respect to the symmetrized harmonics.

$$S\exp\{i2\pi(k_1,x_1) + \dots + i2\pi(k_N,x_N)\},$$

where $S$ is the summetrization operator. Between that Fourier harmonics we choosed [1] the functions

$$u_k = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} \exp\{i2\pi(k,x_j)\}. k \in \mathbf{Z}^3, x \in \mathbf{R}^3.$$

It was proved [1] that any symmetrical Fourier harmonic is a polinomial function from some finite number of the funcions $u_{k_1},\dots,\bar{u}_{k_m}$. That means that we can look for the asymptotic solution to the problem (1) as a composite function of infinite number funtions $u_k$, $\bar{u}_k; k \in \mathbf{P}$; where $\mathbf{P}$ is the subset of $\mathbf{Z}^3 \setminus 0$ such that $\mathbf{P} \cup (-\mathbf{P}) = \mathbf{Z}^3 \setminus 0$ and $\mathbf{P} \cap (-\mathbf{P}) = \phi$. In reality we'll use the functions of the form

$$\exp\left(\frac{\sqrt{N}}{\epsilon}\sum_{k\in p}\beta_k u_k \bar{u}_k\right) Q(\ldots u_k, \bar{u}_k, \ldots) \qquad (3)$$

where $\beta_k <, \Sigma|\beta_k| < \infty$ and $Q$ are some polinomial in finite numbers of functions $u_k, \bar{u}_k$. In the following we set

$$\beta_k = \frac{\epsilon}{2\sqrt{N}}\left(1 - \sqrt{1 + \frac{2\nu_k N}{k^2 4\pi^2 \epsilon^2}}\right).$$

If is proved that at the functions $f(\ldots u_k, \bar{u}_k, \ldots)$ the problem (1) is reduced in the main term to the spectral problem of the sum of non interacting harmonic oscillators.

With the quasimodes (3) we constructed the asymptotics of some spectral series to the problem (1) and proved for the minimal eigenvalue of the problem (1) double sided estimate

$$\frac{\nu_0 N^2}{2} - \frac{N}{2}\sum_{||k|=1}^{\infty} \nu_k + 2\epsilon\sqrt{N}c^2 < \lambda_0 \leq$$

$$\frac{\nu_0 N^2}{2} - \frac{N}{2}\sum_{|k|=1} \nu_k - 8\epsilon\sqrt{N}\pi^2 \Sigma\beta_k K^2$$

with some constant $c$. The details will be given at the talk.

**References.**


1. 1. Valeri V. Kucherenko "Semiclassic for the $N$ partcles Schrödinger equation with binary potential" International conference "Differential equation and Related Topics". Moscow may 22-27, 2001. Book of abstracts, p. 220-221.


## 30 L. Kuciuk: On quantum smaradache paradoxes

A survey on the below intriguing and funny quantum Smarandache paradoxes is presented, paradoxes that are interconnected with many aspects of quantum physics:

1) Sorites Paradox (associated with Eubulides of Miletus (fourth century B.C.):

Our visible world is composed of a totality of invisible particles.

a) An invisible particle does not form a visible object, nor do two invisible particles, three invisible particles, etc. However, at some point, the collection of invisible particles becomes large enough to form a visible object, but there is apparently no definite point where this occurs.

b) A similar paradox is developed in an opposite direction. It is always possible to remove a particle from an object in such a way that what is left is still a visible object. However, repeating and repeating this process, at some point, the visible object is decomposed so that the left part becomes invisible, but there is no definite point where this occurs.

Generally, between ¡A¿ and ¡Non-A¿ there is no clear distinction, no exact frontier. Where does ¡A¿ really end and ¡Non-A¿ begin? One extends Zadeh's "fuzzy set" term to the "neutrosophic set" concept.

2) Uncertainty Paradox: Large matter, which is under the 'determinist principle', is formed by a totality of elementary particles, which are under Heisenberg's 'indeterminacy principle'.

3) Unstable Paradox: Stable matter is formed by unstable elementary particles (elementary particles decay when free).

4) Short Time Living Paradox: Long time living matter is formed by very short time living elementary particles.

## 31 M. Kudryavtsev: The Cauchy problem for the Toda lattice with a class of non-stabilized initial data

We consider the Cauchy problem for the Toda lattice in the case when the corresponding $L$-

operator is a Jacobi matrix with bounded elements, whose spectrum of multiplicity 2 is separated from its simple spectrum and contains an interval of absolutely continuous spectrum. A new type of spectral data, which are analogous for scattering data, is introduced for this matrix. An integral equation that allows us to reconstruct the matrix from this spectral data is obtained. We use this equation to solve the Cauchy problem for the Toda lattice with the initial data that are not stabilized.

## 32 E. Ley-Koo: Variational theorem for the solution of the many-electron relativistic quantum theory

With C.F. Bunge and R. Juregui. The exact solutions of Diracs equation for the free-electron and for hydrogen-like atoms are reviewed with emphasis on the existence of the positive and negative energy states. The problems associated with the solution of the Dirac-Coulomb equation for many-electron atoms, including the dissolution in the continium, first analyzed by Brown and Ravenhall in 1951, and the projection into positive energy-states in the no-pair Hamiltonian, advocated by Sucher, are also reviewed. The Goldman-Drake variational solution of the Dirac equation for the hydrogen-like atoms including both positive and negative energy solutions is also analyzed. In this contribution a variational theorem is formulated and proven for the solutions of the Dirac-Coulomb equation for many electron-atoms including positive and negative energy states.

## 33 E. H. Lieb: Stability of a model of relativistic quantum electrodynamics

The relativistic "no pair" model of quantum electrodynamics uses the Dirac operator, $D(A)$ for the electron dynamics together with the usual self-energy of the quantized ultraviolet cutoff electromagnetic field $A$ — in the Coulomb gauge. There are no positrons because the electron wave functions are constrained to lie in the positive spectral subspace of some Dirac operator, $D$, but the model is defined for any number, $N$, of electrons, and hence describes a true many-body system. In addition to the electrons there are a number, $K$, of fixed nuclei with charges $\leq Z$. If the fields are not quantized but are classical, it was shown earlier that such a model is always unstable (the ground state energy $E = -\infty$) if one uses the customary $D(0)$to define the electron space, but is stable ($E > -\text{const.}(N + K)$) if one uses $D(A)$ itself (provided the fine structure constant $\alpha$ and $Z$ are not too large). This result is extended to quantized fields here, and stability is proved for $\alpha = 1/137$ and $Z \leq 42$. This formulation of QED is somewhat unusual because it means that the electron Hilbert space is inextricably linked to the photon Fock space. But such a linkage appears to better describe the real world of photons and electrons. (joint work with Michael Loss)

## 34 J. R. Mercado-Escalante: Inverse problem for the similarity exponents

The inverse problem of diffusion is solved by fractal methods and the groups analysis of the differential equations. In the multidimensional case, the direct problem of the Boussinesq's equation for the porous medium will be solved. The infinitesimal generator of the groups of the equation differential are found. The mutual relation between the infinitesimals and the dimension space is analysed. The group analysis of auxiliary and especial equations for the porous medium is made, regarding ordinary differential equations. The reduce equation is obtained. The inverse problem for the exponents of the similarity variables is solved. The similarity variables are defined and we find the self-similar solutions. An alternative presentation is conformed, with a condition of normalization to introduce an initial condition. The results to the mathematical models are applied the horizontal drains of agriculture soil, the vertical drains soil and the irrigation by drops.

## 35 B. Mityagin: Counting Lemma for $1D$ periodic Dirac Operator

Let $L = iJ\frac{d}{dx} + V$, $J = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $V = \begin{pmatrix} 0 & p \\ q & 0 \end{pmatrix}$, be Dirac operator with periodic $L^2$-potential $V$, $V(x+1) = V(x)$. For any weight sequence $B(k)$, $B(k) = B(-k) > 0$, $B(k) \uparrow \infty$. We define the space of potentials

$$H(V) = \{V :\ \|V\|_B^2 = \sum(|p_k|^2 + |q_k|^2)B^2(k) < \infty\},$$

where $p_k$ and $q_k$ are the Fourier coefficients of the functions $p$ and $q$.

We consider boundary conditions (bc) of three types: $Per^+ : F(1) = F(0)$; $Per^- : F(1) = -F(0)$; $Dir :\ f(0) = g(0),\ f(1) = g(0)$, and denote by $L_{bc}$ the closure of $L_{|H^1_{bc}}$ in $L^2$.

Put

$$\Pi(X, Y) = \{z \in \mathbf{C}^1 |\ |Re\, z| \le X,\ |Im\, z| \le Y\}$$

and

$$D(a, \delta) = \{z \in \mathbf{C}^1 |\ |z - a| < \delta\}.$$

$L$ is well-defined on vector functions $F = \begin{pmatrix} f \\ g \end{pmatrix}$, $f, g \in H^1$.

**Theorem 1** *(Counting Lemma) There exists an absolute constant $K$, and dependent on $(B_k)$ integer valued function $f(u)$, and a sequence $\delta = (\delta_n)$, $\delta_n \downarrow 0$, such that*

$$\sigma(L_{bc}) \subset \Pi(X, Y) \cup \bigcup_{|n|>N} D(\pi n; \delta_n \|V\|_{H(B)}),$$

*where*

$$X = K(1 + \|V\|_2)^2, \quad Y = \pi(N + 1/2), \quad N = f(\|V\|_{H(B)}).$$

**Theorem 2** *Normalized eigen-functions of $L_{bc}$ are uniformly $L^\infty$-bounded. Moreover, the same is true for all normalized functions in Riesz-subspaces $E_n = Im\, P_n$,*

$$P_n = \frac{1}{2\pi i} \int_{|z-\pi n|=\delta_n} (z - L_{bc})^{-1} dz, \quad |n| > N.$$

These results remain valid if potential $V$ is in the Banach-Orlicz space $L \log^2 L$ and $H(B)$ is substituted by a rearrangement-invariant space $E$ of ¿$L^1$-functions with compact imbedding $E \to L \log^2 L$.

## 36 P. I. Naumkin:Large time asymptotics for quadratic nonlinear Schrödinger equations in one space dimension

We consider the Cauchy problem for the nonlinear Schrdinger equations with quadratic nonlinearities of derivative type in one space dimension. We are interested in the global existence and large time asymptotic behavior of solutions of the Cauchy problem. Heuristically the quadratic type nonlinearities should be considered as sub critical from the point of view of large time behavior, since the quadratic nonlinearities decay in the uniform norm more slowly than the linear terms in the Schrdinger equation. However as we show, due to the special oscillating structure of the nonlinear term the asymptotics behavior of solutions have a modified character. Our approach is similar to the method of normal forms of Shatah.

## 37 F. Nicoleau: A scattering inverse problem with the Aharonov-Bohm effect

We study an inverse scattering problem for Schrdinger operators with a convex obstacle and with an electromagnetic field in order to study the Aharonov-Bohm effect. In dimension greater than 3, we show that the electric and the magnetic field are uniquely determined. In the two dimensional case, some obstruction appears, based on a quantification of the magnetic flux. The same approach can be used to study an inverse scattering problem for a pair of Hamiltonians $(H(h), H_0(h))$, where $H_0(h) = -h^2\Delta$ and $H(h) = H_0(h) + V$,$V$ is a short-range potential. We show that in dimension $n \geq 3$, the scatterings operators $S(h)$, $h \to 0$, which are localized near a fixed energy $\lambda > 0$ determine the asymptotics of the potential $V$ at infinity.

## 38 R. H. Parmar: Super symmetric quantum mechanics with vector super potential

With P. C. Vinodkumar. SUSYQM deals usually with one-dimensional systems [1]. Here we define the SUSY charge operators in three dimensional vector notations and define their scalar products to represent the Hamiltonian. It can then easily be generlized for other higher dimensions.

We define here a general SUSY vector potential $\vec{\Phi}$ as $-\vec{\nabla}\ \psi_0/\psi_0$ such that the SUSY charge operators can be written as,

$$\vec{Q^+} = \frac{1}{\sqrt{2}}\left(-\vec{\nabla} - \frac{\vec{\nabla}\psi_0}{\psi_0}\right)\ ;\ \vec{Q^-} = \frac{1}{\sqrt{2}}\left(\vec{\nabla} - \frac{\vec{\nabla}\psi_0}{\psi_0}\right) \tag{4}$$

Their scalar products according to SUSYQM [1] provide

$$\vec{Q^+}\cdot\vec{Q^-} = H^0 = \frac{1}{2}\left[-\nabla^2 + \vec{\nabla}\cdot\left(\frac{\vec{\nabla}\ \psi_0}{\psi_0}\right) + \left(\frac{\vec{\nabla}\ \psi_0}{\psi_0}\right)^2\right] \tag{5}$$

and

$$\vec{Q^-}\cdot\vec{Q^+} = H^1 = \frac{1}{2}\left[-\nabla^2 + \left(\frac{\vec{\nabla}\ \psi_0}{\psi_0}\right)^2 - \vec{\nabla}\cdot\left(\frac{\vec{\nabla}\ \psi_0}{\psi_0}\right)\right] \tag{6}$$

Then the two pair potentials $V^0$ and $V^1$ correspond to $H^0$ and $H^1$ are

$$V^0(\vec{r}) = \frac{1}{2}\left[\vec{\Phi}\cdot\vec{\Phi} - \vec{\bigtriangledown}\cdot\vec{\Phi}\right] \tag{7}$$

and

$$V^1(\vec{r}) = \frac{1}{2}\left[\vec{\Phi}\cdot\vec{\Phi} + \vec{\bigtriangledown}\cdot\vec{\Phi}\right] \tag{8}$$

For a solenoidal SUSY potential $\vec{\Phi}$, $H^1 = H^0$. This condition becomes a special case of the SUSYQM with vector potential.

Expressing the Hamiltonian in terms of Spherical polar co-ordinates, we get

$$H^0 = \frac{1}{2}\left( - \bigtriangledown^2 + \Phi_r^2 \ + \ \Phi_\theta^2 \ + \ \Phi_\varphi^2 \ - \ \Phi_r^{'} \ - \ \frac{2}{r}\Phi_r \ - \ \frac{\Phi_\theta^{'}}{r} \ - \ \frac{\Phi_\theta}{r\ tan\theta} \ - \ \frac{\Phi_\varphi^{'}}{r\ sin\theta}\right) \tag{9}$$

and

$$H^1 = \frac{1}{2}\left( - \bigtriangledown^2 + \Phi_r^2 \ + \ \Phi_\theta^2 \ + \ \Phi_\varphi^2 \ + \ \Phi_r^{'} \ + \ \frac{2}{r}\Phi_r \ + \ \frac{\Phi_\theta^{'}}{r} \ + \ \frac{\Phi_\theta}{r\ tan\theta} \ + \ \frac{\Phi_\varphi^{'}}{r\ sin\theta}\right) \tag{10}$$

Now, if $\vec{\Phi}$ is independent of $\theta$ and $\varphi$ and depend only on $r$, the Hamiltonian become,

$$H^0 = \frac{1}{2}\left( - \bigtriangledown^2 + \Phi_r^2 \ - \ \Phi_r^{'} \ - \ \frac{2}{r}\Phi_r\right); H^1 = \frac{1}{2}\left( - \bigtriangledown^2 + \Phi_r^2 \ + \ \Phi_r^{'} \ + \ \frac{2}{r}\Phi_r\right) \tag{11}$$

The advantage here is that the super potential $\phi_r$ does not depend on the angular momentum unlike in the regular SUSYQM for spherical symmetric cases [2]. An additional term of $\pm\ 2\Phi_r/r$ also appear in the Hamiltonian. For a choice of $\vec{\Phi} = \vec{r}$ leads to the usual harmonic oscillator problem in SUSYQM. Other choices of $\vec{\Phi}$ and their solutions will be discussed.

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## 39 D. Pearson: Recent developments in value distribution theory for Schrödinger operators

The theory of value distribution for boundary values of Herglotz functions is closely linked to spectral properties of Schrödinger operators on the half line.The theory is used to provide a large distance asymptotics for solutions of the Schrödinger equation, and throws light on such questions as the existence of singular and absolutely continuous spectra and estimates of the Weyl-Titchmarsh m-function for sparse potentials. Some recent developments of this work are presented and lead to a characterization of asymptotics for the support of ac measures.

## 40 G. S. Pogosyan: Coulomb-oscillator duality in spaces of constant curvature

With E. G. Kalnins, G. and W. Miller, Jr. We have constructed a series of mappings $S_{2C} \to S_2, S_{4C} \to S_3$ and $S_{8C} \to S_5$, that are generalize those well known from the Euclidean space Levi-Civita, Kustaanheimo-Stiefel and Hurwitz transformations. It have shown, that as in case of flat space, these transformations permit one to establish the *correspondence* between the Kepler-Coulomb and oscillator problems in classical and quantum mechanics for the respective dimensions. Using these transformations the quantum Coulomb system on the two-, three- and five-dimensional sphere are completely solved, including eigenfunctions with correct normalization constant and energy spectrum.

## 41 R. Quezada: A class of non-conservative quantum dynamical semigroups

Quantum dynamical semigroups (qds) appear as solutions of the so called master equation which is a generalization of the Schrodinger equation in the Heisenberg representation with noise terms; that is the reason why a qds describes the irreversible (dissipative, nonunitary) evolution of a quantum system. In the theory of stochastic processes the master equation is a noncommutative extension of the backward Kolmogorov equation. The conservativity (Markovianity or unitality) of a qds is related with the non-explosion of a stochastic process and it is useful to assure the uniqueness of the minimal solution of a master equation. This property has been intensively studied in the recent years. Much less attention has received the class of non-conservative qds, nevertheless its study is important from both the mathematical point of view as well as for applications. Our main aim in this work is to study a class of non-conservative qds that naturally appear when a necessary condition for conservativity is not satisfied.

## 42 V. Rabinovich: Pseudodifferential operators with analytic symbols and estimates for eigenfunctions of Schrödinger operators

We study the behaviour of eigenfunctions of the Schrödinger operator $-\Delta + v(x)$ with potentials having power, exponential or super-exponential growth at infinity and discontinuities on manifolds in $\mathbf{R}^n$. We use a connection between the domain of analyticity of the main symbol $\left(|\xi|^2 + v(x)\right)^{-1}$ of the parametrix $-\Delta + v(x)$ at infinity or near singularities of $v(x)$ and the behaviour of eigenfunctions at infinity or near singularities of potentials. Our approach is based on a general calculus of pseudodifferential operators with analytic symbols.

## 43 R. del Río : Inverse spectral results for Dirac systems

We generalize results of Hochstadt and Lieberman for Dirac Systems. From partial knowledge of the potential and partial knowledge of the spectra it is shown that the rest of the potential function is uniquely determined. An important ingredient in our strategy is the link between the rate of growth of an entire function and the distribution of its zeros. This is joint work with Benoit Grebert.

## 44 D. Robert: Long time propagation for quantum observables and coherent states in the semiclassical régime

In this lecture we review some recent results obtained for solutions of time dependent Schrödinger equations in the semiclassical régime ($\hbar \to 0$). We consider as well propagation of quantum observables (Heisenberg picture) and propagation of Gaussian coherent states. The main problems discussed here concern the validity of the asymptotic expansions in $\hbar$ for large times. For general systems (in particular for classical chaotic systems) we prove that the Ehrenfest time $T_\hbar = c \log\left(\frac{1}{\hbar}\right)$, where $c > 0$ depends on unstability Lyapounov exponents, is a true time limitation for the semiclassical régime. On the opposite side, for integrable systems this time is much longer (at leat $o(\hbar^{-1/2})$).

Furthermore for analytic or Gevrey smooth Hamiltonians we get analytic or Gevrey estimates for the corresponding semiclassical asymptotic expansions and consequently exponentially small remainder estimates.

## 45 Gert Roepstorff: On a Class of anomaly free gauge theories

This is a report on a calculation of the anomaly coefficients for the reducible representation $\theta$ of a Lie algebra Lie $G$ on $\bigwedge \mathbf{C}^n$. Assuming that $G \subset U(n)$ where $n \geq 2$, the representation $\theta$ is obtained from lifting the action of $U(n)$ on $\mathbf{C}^n$ to the exterior algebra. The coefficients vanish provided $G \subset SU(n)$ and $n \neq 3$. The singular role of the group $SU(3)$ is emphasized.

## 46 M. B. Ruskai: Comment on efficiency of adiabatic quantum algorithms

It is argued that a proposed algorithm for adiabatic quantum computation can not solve an NP-complete problem faster than Grover's algorithm, which requires $O(2^{n/2})$ time. There has been considerable interest in a proposed scheme for adiabatic quantum computation [1, 2] and speculation that it may provide a mechanism for effcient solution of hard problems. However, the latter is based on extrap olations of numerical simulations for model problems, particularly the "exact cover" problem, with $n < 20$. The adiabatic quantum algorithm is designed to take the ground state of an initial Hamiltonian H0 to that of a final Hamiltonian H1 using a linear interpolating Hamiltonian of the form H(t)=(1- t)H0 + tH1 . The efficiency of the adiabatic approximation has been shown to be $O(1/g2min)$ where g min denotes the minimum energy gap g(t)=E 1 (t) -E 0 (t) between the ground and first excited states of H(t). The "computational basis" —k 1...k n consists of products of eigenfunctions of the Pauli matrix z . Fahri, et al use the initial Hamiltonian H 0 = j x (j) whose eigenstates can be obtained by applying Hadamard transforms to the states —k 1...k n , yielding states of the form 2 -n/2 k 1 ...k n 1 —k 1...k n (with all signs +1 for the ground state). Fahri, et al define a nonnegative final Hamil tonian H 1 which is diagonal in the computational basis and has the solution of the Exact Cover problem as its ground state, i.e., they encode the Exact Cover problem in the computational basis. Grover [3, 4] showed how to encode searching an unordered list into the computational basis via the construction of a unitary operator G whose effect is simply to multiply the target state —$j_1 \ldots j_n$ by 1 and all others by +1. (The state —$j_1 \ldots j_n$ need not be known in advance, only a method of identifying it.) If we now let $H_2 = GH_1$, we obtain a Hamiltonian [5] identical to $H_1$ except that the eigenvalue of the target state —$j_1 \ldots j_n$ is multiplied by 1. Because $H_1$ is nonnegative, $H_2$ has exactly one negative eigenvalue so that its ground state is now the target state —$j_1 \ldots j_n$. Applying the adiabatic algorithm to the modified interpolating Hamiltonian $H_2(t)=(1\ t)H_0+tH_2$ should take the ground state of $H_0$ to the target state. Moreover, the only effect on the final Hamiltonian is to move one excited state below the previous ground state, without decreasing the final energy gap g(1). There is little reason to expect this to significantly affect $g_{min}(t)$; therefore, one would expect an adiabatic search to be as effcient as the solution of the problem encoded in $H_1$. Thus, if Fahri, et al's projection of an efficiency of $O(n^2)$ is correct, one would expect an efficiency of $O([\log N]^2)$ for the adiabatic search of a list of $N=2^n$ items described above. More generally, an adiabatic efficiency of $O(n^p)$ would imply an adiabatic search efficiency for $N=2^n$ items of $O([\log N]^p)$. However, this contradicts earlier arguments [6, 4] that a quantum computer could not search an unordered list in better than $O(\ N)$ time. The validity of the adiabatic approximation depends on the existence of an eigenvalue gap (which should follow from the uniqueness of the ground state [7]), but the efficiency of the proposed algorithm, also requires that the gap decrease sufficiently slowly as n increases. This has not been established. In standard applications of the adiabatic approximation, it is reasonable to expect that perturbations of the form $(H_1\ H_0)$ mix only a few lowlying states. However, the proposed algorithm transforms the initial state to a final (product) state which is an evenly weighted superposition of all $2^n$ excited states of $H_0$. This suggests that a perturbation expansion will converge slowly which implies that g(t) must decrease rapidly or, alternatively, that many small time steps are needed to mix the required 2 n states.

Acknowledgement: It is a pleasure to thank Professors Charles Bennett, David DiVicenzio, George Hagedorn and Christopher King for helpful discussions and comments. An expanded discussion of some of the issues raised here will be posted on the quantph preprint server. This research was partially supported by the National Security Agency and Advanced Research and Development Activity under Army Research Office contract number DAAG559810374 and by the National Science Foundation under Grant number DMS0074566.

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5. Because G and H 1 are both diagonal in the computational basis, they com mute and H 2 is selfadjoint.

. 6 C.H. Bennett, E. Bernstein, G. Brassard and U. Vazirani "Strengths and Weaknesses of

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7. As will be discussed in a paper to be posted at arXiv.org/quantph, there are some serious questions about Fahri, et al's use of the noncrossing rule to establish uniqueness of the ground state. However, for the Hamiltonians in question, this can be done by a standard PerronFrobenius argument.

## 47 A. Rybkin: On the generalized spectral shift function for one-dimensional Schrodinger operators with slowly decaying potentials and trace formulas

In our talk we are concerned with self-adjoint Schrodinger operators on the line. It is well-known that if the potential is summable then Krein's spectral shift function exists (as well as the scattering matrix) and a variety of trace formulas can be described. The situation is different if the potential is decaying but slowly. We concentrate on the case when the potential is squire integrable. We intend to discuss the properties of a certain generalized spectral shift function and describe analogs of trace formulas in this setting.

## 48 A. Sachetti: Tunneling instability for a double well Schroedinger operator with a non-linear perturbation

I consider a non-linear perturbation of a symmetric double-well potential as a model for molecular localization. In the semiclassical limit, I prove the existence of a critical value of the perturbation parameter giving the destruction of the beating effect.

## 49 R. Schrader: Statistical ensembles and density of states

In our talk we will give a brief exposition of joint approach with V.Kostrykin to the statistical theory of small systems using the concept of integrated density of states familiar from the theory of random Schrödinger operators. The idea to consider the thermodynamics for systems with a small number of particles is about 30 years old with one of the first articles due to Thirring. It has found applications in nuclear physics and in astrophysics. The present approach covers both the classical and the quantum case and thus allows for a comparison. A microcanonical ensemble may be defined provided the integrated density of states is twice differentiable. The structure of the van Hove singularities in the case of one particle moving in a periodic potential in 3 dimensions implies that the corresponding integrated density of states for n noninteracting particles is [(3n-1)/2] times differentiable. The case of random potentials and of an additional interaction between the particles remains an open problem. In analogy a canonical ensemble may be defined and may be compared with the microcanonical ensemble in a way familiar from the standard formulation. The classical case allows for a new example of a system giving rise to a negative specific heat. In the quantum case and for the microcanonical ensemble the relation between the integrated density of states and the scattering phase shift density - obtained in collaboration with V. Kostrykin - combined with a conjectured high enery behavior of the phase shift gives rise to an explicit form of the (mean) energy-temperature relation in this high energy regime.

## 50 B. Schumayer: Painlevé test of coupled nonlinear Schrödinger equations and Bose-Einstein condensates

Nonlinear dynamical evolution equations are often employed in physics, especially in field theories. Historically, the detailed examination of the KdV equation shows that this type of equations possesses special solutions, e.g., solitary waves, solitons. Furthermore, these so called totally integrable equations, like the KdV itself, or the nonlinear Schrödinger equation, or its variant, the Gross-Pitaevskii equation, can be solved by the method of inverse scattering transformation. In 1978 Ablowitz, Ramani and Segur [1] made a conjecture: if an equation passes the Painlevé-test it can be solved by inverse scattering transformation.

In our work we apply [2] the Painlevé test to the coupled nonlinear Gross-Pitaevskii equations describing a pile of weekly interacting atomic particles belonging to the same Bose-Einstein condensed state. We show that the system of equations becomes integrable for special external potentials depending on system parameters such as masses and interaction strengths. We derive a formula for classifying the various cases in terms of a single integer constant $m$. Our example with $m = 2$ turns out to agree with an earlier investigation carried out by Clarkson [3] for the uncoupled case. Finally, we derive [4] also a transformation between the solutions of the coupled nonlinear Schrödinger equations and those of the coupled Gross-Pitaevskii equations. Our transformation ansatz and general formulas enable direct use of the highly evolved soliton solutions of the coupled nonlinear Schrödinger equations in recent two-component Bose-Einstein condensate experiments.

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## 51 R. Seiringer: Gross-Pitaevskii Theory of the Rotating Bose Gas

We study the Gross-Pitaevskii functional for a rotating two-dimensional Bose gas in a trap. We prove that there is a breaking of the rotational symmetry in the ground state; more precisely, for any value of the angular velocity and for large enough values of the interaction strength, the ground state of the functional is not an eigenfunction of the angular momentum. This has interesting consequences on the Bose gas with spin; in particular, the ground state energy depends non-trivially on the number of spin components, and the different components do not have the same wave function. For the special case of a harmonic trap potential, we give explicit upper and lower bounds on the critical coupling constant for symmetry breaking.

## 52 E Sere: A ground state for the Dirac-Fock model

The Dirac-Fock equations are the relativistic analogue of the well known Hartree-Fock equations. They are used in quantum chemistry, and yield results on the inner-shell electrons of heavy atoms. We give some existence results for the solutions of the Dirac-Fock equations, and we study the convergence of these solutions in the nonrelativistic limit. As a byproduct, we define a notion of ground state for the Dirac-Fock model, valid when the speed of light is large enough. This is joint work with M.J. Esteban.

## 53 H. Siedentop: The Energy of relativistic one-electron atoms

Joint work with Edgardo Stockmeyer and Raymond Brummelhuis. Relativistic particles have been described by Hamiltonians that are the quantization of the relativistic symbol $\sqrt{p^2+m^2} - Z\alpha/|x|$.

This Hamiltonian has been investigated by Herbst and Weder. The critical coupling constant, however, turns out to be $\alpha Z \leq 2/\pi$ which does not cover all elements. In the present talk we will present a model going back to Douglas and Kroll, and Jansen and Heß. We will show that the energy of this Hamiltonian used in quantum chemistry which describes all known elements numerically to high accuracy is bounded from below for $\alpha Z \leq \gamma_c$ where $\gamma_c$ slightly exceeds 1. This fact will allow to construct a Hamiltonian which is self-adjoint.

## 54 F. Sobieckzky: When do digital images fragment into infinitely many segments

The image segmentation problem is the question of how to divide the domain of mapping into subsets which fulfill some homogeneity criterion. An analytical image segmentation method is proposed by which such criteria are investigated as to whether they separate digital images into infinitely many segments, as the dimension of the images' representation (and thereby the degree of detail) is increased to infinity. In order to describe the quantized structure of the discrete (digital) image data, the dynamics given by a Schroedinger Operator of a quantum spin system is considered. Using von Neuman's ergodic theorem, the procedure aimes at finding the orthogonal projectors onto characteristic subsets of a lattice, which contain a single given lattice point. Segments are identified with the invariant subsets of the underlying non-ergodic dynamical system. Key-words: Quantized Region Growing, Stability of Quantum Spin Systems, Hilbert Space methods and Image Segmentation, finite Magnetization.

## 55 S. Sontz: New results in reverse inequalities

Based on earlier work of Carlen on reverse hypercontractivity and by the author on reverse log-

Sobolev inequalities (both in the Segal-Bargmann space), we have recently obtained generalizations of these results to a certain class of complex manifolds. This is joint work with F. Galaz-Fontes and L. Gross.

## 56 P. Stovicek: Weakly regular Floquet Hamiltonians with pure point spectrum

We study the Floquet Hamiltonian $-i\partial_t + H + V(\omega t)$, acting in $L^2([\,0,T\,],\mathcal{H},dt)$, as depending on the parameter $\omega = 2\pi/T$. We assume that the spectrum of $H$ in $\mathcal{H}$ is discrete, $\mathrm{Spec}(H) = \{h_m\}_{m=1}^{\infty}$, but possibly degenerate, and that $t \mapsto V(t) \in \mathcal{B}(\mathcal{H})$ is a $2\pi$-periodic function with values in the space of Hermitian operators on $\mathcal{H}$. Let $J > 0$ and set $\Omega_0 = [\,\frac{8}{9}J, \frac{9}{8}J\,]$. Suppose that for some $\sigma > 0$ it holds true that $\sum_{h_m > h_n} \mu_{mn}(h_m - h_n)^{-\sigma} < \infty$ where $\mu_{mn} = (\min\{M_m, M_n\})^{1/2} M_m M_n$ and $M_m$ is the multiplicity of $h_m$. We show that in that case there exist a suitable norm to measure the regularity of $V$, denoted $\epsilon_V$, and positive constants, $\epsilon_\star$ and $\delta_\star$, with the property: if $\epsilon_V < \epsilon_\star$ then there exists a measurable subset $\Omega_\infty \subset \Omega_0$ such that its Lebesgue measure fulfills $|\Omega_\infty| \geq |\Omega_0| - \delta_\star \epsilon_V$ and the Floquet Hamiltonian has a pure point spectrum for all $\omega \in \Omega_\infty$.

## 57 M. Tater: Scattering by a slab

The model consists of an infinite number of point interactions distributed periodically on a finite number of parallel planes. Exact formulae for the transmission coefficients are given. Numerical behaviour is presented, first in the case the incident plane wave has a wave vector perpendicular to the planes and then for some tilted directions. For some values of the interaction parameter the reflection coefficient is close to one for a relatively large set of incident wavelengths.

## 58 A. Tip: Maxwell with a touch of Schrödinger

We consider the macroscopic Maxwell's equations for a linear, in general absorptive, dielectric. Such systems are of interest for optical devices, in particular photonic crystals, dielectrics with a spatial periodicity. Without absorption and dispersion, i.e. if the electric permeability $\varepsilon(\mathbf{x})$ is a function of position only, it is straightforward to set up a unitary time evolution scheme and the spectral structure of the generator then gives information about a number of the system's properties. In particular for a photonic crystal, where $\varepsilon(\mathbf{x}+\mathbf{a}) = \varepsilon(\mathbf{x})$, $\mathbf{a}$ being the lattice vector, a band structure emerges and there may be band gaps. The latter can become of great physical interest for the construction of photonic devices, such as solar cells, solid state lasers and, possibly, quantum computers. The reason is that excited states of atoms, embedded in such crystals, can become stable if their transition frequency falls in a band gap. Then there are no field modes available to transport away the atomic excitation energy.

However, it is difficult to construct such photonic crystals with gaps in the optical region and this has led to the study of absorptive devices, which seem to be more promising. But now there is a difficulty since $\mathbf{D}$ in Maxwell's equations

$$\partial_t \mathbf{D}(\mathbf{x},t) = \partial_{\mathbf{x}} \times \mathbf{B}(\mathbf{x},t), \quad \partial_t \mathbf{B}(\mathbf{x},t) = -\partial_{\mathbf{x}} \times \mathbf{E}(\mathbf{x},t), \tag{12}$$

is related to $\mathbf{E}$ through a time convolution

$$\mathbf{D}(\mathbf{x},t) = \mathbf{E}(\mathbf{x},t) + \int_{-\infty}^{t} ds \chi(\mathbf{x},t-s)\mathbf{E}(\mathbf{x},s), \tag{13}$$

and (1) does not define a unitary time evolution.

However, it is possible to introduce two auxiliary fields $\mathbf{F}_{2,4}(\mathbf{x},\lambda,t)$, $\lambda\in[0,\infty)$, such that

$$\mathbf{F}(t)=\begin{pmatrix}\mathbf{F}_1(t)\\ \mathbf{F}_2(t)\\ \mathbf{F}_3(t)\\ \mathbf{F}_4(t)\end{pmatrix}=\begin{pmatrix}\mathbf{E}(t)\\ \mathbf{F}_2(t)\\ \mathbf{B}(t)\\ \mathbf{F}_4(t)\end{pmatrix}, \tag{14}$$

satisfies a unitary time evolution

$$\partial_t\mathbf{F}(t)=-i\mathsf{K}\mathbf{F}(t) \tag{15}$$

in an appropriate Hilbert space of which the norm is proportional to the conserved energy [1], the latter being an extension of the conserved electromagnetic energy

$$\mathcal{E}=\frac{1}{2}\int d\mathbf{x}\{\mathbf{E}(\mathbf{x},t)^2+\mathbf{B}(\mathbf{x},t)^2\}. \tag{16}$$

In an absorptive photonic crystal we have $\chi(\mathbf{x}+\mathbf{a},t)=\chi(\mathbf{x},t)$, leading to a discrete translational invariance of the selfadjoint generator $\mathsf{K}$, thus allowing a Bloch decomposition as in the Schrödinger case. But in the zero order case, where $\mathbf{F}_{2,4}$ are decoupled from the physical fields $\mathbf{E}$ and $\mathbf{B}$, the generator for the subset

$$\begin{pmatrix}\mathbf{F}_2\\ \mathbf{F}_4\end{pmatrix} \tag{17}$$

has the full positive real axis as its spectrum and this excludes the possibility of $\mathsf{K}$ having spectral gaps. But we are only interested in the physical fields and it is sufficient that their Fourier transforms relative to the time variable, $\tilde{\mathbf{F}}_{1,3}(\mathbf{x},\omega)$, have gaps.

We shall present a precise definition of a band gap along these lines and discuss the spectral properties of $\mathsf{K}$ for the photonic crystal case. Using a dilatation transformation on the $\lambda$-variable, above, we find that the complex-dilated $\mathsf{K}(\zeta)$ has spectrum consisting of areas in the lower complex plane. The analysis, which uses the usual relative compactness arguments, is a good deal more complicated than in the Schrödinger case, due to the presence of an infinite-dimensional null space. In general band gaps no longer occur [2]. We shall also show the results of calculations of the dilated spectrum for a two-dimensional case, confirming these results [3].

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## 59 J. Toloza: Exponentially Small Error Estimates of Quasiclassical Eigenvalues

We study the behavior of truncated Rayleigh-Schrödinger series for the low-lying eigenvalues of the time-independent Schrödinger equation, in the semiclassical limit $\hbar \searrow 0$. We prove that, under certain hypotheses on the potential $V(x)$ and for small $\hbar > 0$, the perturbation series for the eigenvalues admits an exponentially accurate truncation. That is, the difference between the truncated series and the actual eigenvalue can be made smaller than $\exp(-C/\hbar)$ for a positive constant $C$. An analogous statement is shown for the corresponding eigenfuctions.

## 60 A. Turbiner: Canonical Discretization (discretization as canonical transformation)

A new realization of the Heisenberg algebra in terms of finite-difference (translation covariant) and discrete (dilatation covariant) operators is presented. It is a realization of deformation type and can be treated as a quantum analogue of canonical trasformations. Fock space formalism which allows a notion of spectral problem is introduced. The formalism provides a natural isospectral connection of differential and difference equations which also leaves polynomial eigenfunctions invariant. (An)harmonic oscillator as well as the Hahn equation are considered in details as examples.

## 61 G. Uhlmann: On the local Dirichlet-to-Neumann map

We will discuss some recent results on the problem of determining a conductivity or a potential by measuring the Dirichlet-to-Neumann map associated to the conductivity equation or the Schroedinger equation by making measurements on part of the boundary.

## 62 C. van der Mee: The Inverse generalized Regge problem

With V. Pivovarchik. Small transversal vibrations of a nonhomogeneous string are described by a boundary value problem which by applying a Liouville transformation is transformed into the Sturm-Liouville problem

$$y''(\lambda,x)+\lambda^2 y(\lambda,x)-q(x)y(\lambda,x) = 0, \tag{18}$$

$$y(\lambda,0) = 0, \tag{19}$$

$$y'(\lambda,a)+(i\alpha\lambda+\beta)y(\lambda,a) = 0, \tag{20}$$

where $\alpha > 0$, $\beta \in \mathbf{R}$, and $q \in L_2(0,a)$ is real-valued. For $\alpha = 1$ and $\beta = 0$ we obtain the so-called

Regge problem. We provide a method of constructing the interval length $a$, the potential $q$ and the two constants $\alpha$ and $\beta$ from its eigenvalues. We give necessary and sufficient conditions on sequences of complex numbers to be the eigenvalues of Eqs. (18)-(20) in the separate cases $0 < \alpha 1$. First, the eigenvalues are shown to be the zeros of an entire function belonging to a generalization of the Hermite-Biehler class and their asymptotic properties are derived. To solve the inverse problem, we employ the given sequence of complex numbers to construct entire functions which are then used to construct a scattering function. The potential $q$ then follows with the help of the usual Marchenko method.

## 63 I. Veselic: Wegner estimate for alloy type models with non negative single site potential of small support

We study spectral properties of Schrödinger operators with a random potential of Anderson or alloy type on $L^2(R^d)$ and their restrictions to finite boxes. We extend and unify W. Kirsch's result on Wegner estimates for non-negative single site potentials with small support. Our estimate is valid for all bounded energy intervals. Subsequently we use recent results on the spectral shift function and establish the Hölder continuity of the integrated density of states on the whole energy axis. Wegner estimates play a key role in an existence proof of pure point spectrum of the considered random Schrödinger operators.

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## 64 C. Villegas Blas: The Bargmann transform, canonical transformations and the Kepler problem

In this talk we describe a Bargmann transform when the configuration space is $\mathcal{S}^2$ or $\mathcal{S}^3$ (the unit spheres in the Euclidean spaces of three and four dimensions respectively). That is, a unitary transformation of $L^2(\mathcal{S}^2)$ or $L^2(\mathcal{S}^3)$ onto a Hilbert space of analytical functions. We show that in the kernel of such a transform, we have a generating function of a canonical transformation which is the classical analog of the Bargmann transform. This canonical transformation involves a complex quadric and it is also related to the regularization of the Kepler problem. We also study a space of analytical functions on the quadric and relate our study with results of V. Bargmann and V. Guillemin. Finally, we identify our Bargmann transform as a coherent states transform.

## 65 V. Vougalter: Pauli operator and Aharonov-Casher theorem for measure valued magnetic fields

We define the two dimensional Pauli operator and identify its core for magnetic fields that are Borel measures. The magnetic field is generated by a scalar potential hence we bypass the usual $\mathbf{A} \in L^2_{loc}$ condition on the vector potential which does not allow to consider such singular fields. We extend Aharonov-Casher theorem for magnetic fields that are measures with finite total variation and we present a counterexample in case of infinite total variation. One of the key technical tools is a weighted $L^2$ estimate on a singular integral operator.

### 66 K. B. Wolf: Finite Quantum/Optical Systems

Hamiltonian systems that satisfy the oscillator axioms with a compact dynamical algebra, will have a finite number of energy levels, and the range of positions will have a finite number of values. Such systems satisfy Schrodinger difference equations, and their eigenfunctions involve finite polynomials (Kravchuk, Meixner, Hahn) times the square root of their orthonormality measure. These systems exhibit coherent states, mixed states, a properly defined compact phase space, and a covariant Wigner quasiprobability distribution. Finite Hamiltonian systems are useful in signal processing (finite optics) since they define fractional Fourier-Kravchuk and Hankel-Hahn transforms on finite data sets, and on pixellated square or round screens. These are a sequence of finite unitary approximants to the Fourier and Hankel (Fourier-Bessel) transforms.

### 67 K. Yajima: The local smoothing property and Strichartz inequality for Schrödinger equations with potentials superquadratic at infinity

With G. Zang. We consider $n$-dimensional Schrödinger equations with potentials which grow super-quadratically at infinity:

$$i\frac{\partial u}{\partial t} = -(1/2)\triangle u + V(x)u, \quad x \in \mathbf{R}^n,\ t \in \mathbf{R}; \quad u(0,x) = u_0(x), \quad x \in \mathbf{R}^n. \tag{21}$$

**Assumption 1** *$V(x)$ is real valued and is of $C^\infty$-class. There exist $m > 2$ and $R > 0$ such that:*

1. *For* $|x| \geq R$, $D_1\langle x\rangle^m \leq V(x) \leq D_2\langle x\rangle^m$, *where* $D_1 \leq D_2$ *are positive constants.*

2. *For* $|\alpha| \geq 2$, $|\partial_x^\alpha V(x)| \leq C_\alpha \langle x\rangle^{m-|\alpha|}$.

Under the assumption, the operator $L : u \mapsto -(1/2)\triangle u + V(x)u$ defined on $C_0^\infty(\mathbf{R}^n)$ is essentially selfadjoint in $L^2(\mathbf{R}^n)$ and the solution in $L^2(\mathbf{R}^n)$ of the initial value problem (21) is given by $u(t,\cdot) = U(t)u_0$ via the unitary group $U(t) = e^{-itH}$ generated by the unique selfadjoint extension $H$ of $L$. We show that the solution $u(t,\cdot)$, nonetheless, is much smoother than $u_0$ and $1/m$ times differentiable at almost all time $t \neq 0$. More precisely, we prove the following theorem. We write $\langle A\rangle = (1+|A|^2)^{\frac{1}{2}}$ for a self-adjoint operator $A$ and $D = (D_1, \dots, D_n)$, $D_j = -i\partial/\partial x_j$. $\|\cdot\|_p$ is the norm of Lebesgue space $L^p(\mathbf{R}^n)$ and $\|\cdot\| = \|\cdot\|_2$, $1 \leq p \leq \infty$.

**Theorem 2** *Let* $V$ *satisfy* Assumption 1. *Let* $T > 0$ *and* $\Psi \in C_0^\infty(\mathbf{R}^n)$. *Then, there exists a constant* $C > 0$ *such that*

$$\left(\int_{-T}^{T} \|\Psi(x)\langle D\rangle^{\frac{1}{m}} e^{-itH} u_0\|_2^2 dt\right)^{\frac{1}{2}} \leq C\|u_0\|, \quad u_0 \in L^2(\mathbf{R}^n). \tag{22}$$

Theorem 2 is sharp in the sense that the exponent $1/m$ in (22) cannot in general be replaced by any larger number. This can be seen by taking the potential $V(x) = \langle x_1\rangle^m + \cdots + \langle x_n\rangle^m$ and the initial state $u_0(x) = e_{i_1}(x_1)\cdots e_{i_n}(x_n)$, where $e_j(x)$ is the $j$-th eigenfunction of the one dimensional Schrödinger operator $-(1/2)(d^2/dx^2) + \langle x\rangle^m$, and by using the well known result on the asymptotic behavior as $j \to \infty$ of $e_j(x)$ for $x$ in a compact set.

The method for proving Theorem 2 produces the following Strichartz type inequality as a byproduct which we believe, however, is much weaker than best possible.

**Theorem 3** *Let $V$ satisfy* Assumption 1. *Let $T > 0$ and let $p \in [2, \infty)$, $\theta \in (2, \infty]$ be such that $0 \leq \frac{2}{\theta} = n\left(\frac{1}{2} - \frac{1}{p}\right) < 1$. Then, for any $\gamma > \frac{1}{\theta}\left(\frac{1}{2} - \frac{1}{m}\right)$ there exists a constant $C > 0$ such that*

$$\left(\int_{-T}^{T} \|e^{-itH}u_0\|_p^\theta dt\right)^{\frac{1}{\theta}} \leq C\|\langle H\rangle^\gamma u_0\|, \quad u_0 \in L^2(\mathbf{R}^n). \tag{23}$$

We shall first review the proof of Theorem 2 and Theorem 3 for the case $V = 0$ and the case $V$ is at most quadratic at infinity, in which case Theorem 2 and Theorem 3 hold with $m = 2$ and $\gamma = 0$ respectively, and explain what are the physical contents of these estimates. We then outline the proofs of Theorem 2 and Theorem 3 for superquadratic potentials following the idea which is suggested by the physical meaning of these estimates.

## 68 K. Yoshitomi: Band spectrum of the Laplacian on a slab with the Dirichlet boundary condition on a grid

We study the band spectrum of the Laplace operator on the slab $\Lambda = \mathbf{R}\times] - 1/2, 1/2[$ with the Dirichlet boundary condition on the grid $\partial\Lambda \cup (a\mathbf{Z} \times (] - 1/2, 1/2[\setminus[-\epsilon, \epsilon]))$. We obtain the asymptotic form of the first band function as $\epsilon$ tends to zero. We further investigate the Laplace operator on $\mathbf{R}^2$ with the Dirichlet boundary condition on a hexagonal grid.

## 69 G. Zhislin: Peculiarities of the structure of the spectrum of many particle hamiltonians of charged systems in a homogeneous magnetic field

Let $H$ be the hamiltonian of the charged system $Z_1$ of $n$ quantum particles with finite masses in a homogeneous magnetic field with the direction of $z$-axes; $H$ is written after the separation of center-of-mass motion (c.m.m.) of the system in the direction of the field. Since the separation of c.m.m. in $(x, y)$-plane is impossible, in order to study the spectrum of the relative motion $Z_1$ Avron, Herbst and Simon suggested to study the spectrum of the restriction $H(P)$ of the operator $H$ of one component of pseudomomentum $Z_1$ eigenspace, corresponding to arbitrary fixed eigenvalue $P$ of this component. They discovered the essential spectrum $s_e\big(H(P)\big)$ of $H(P)$, but the structure of the discrete spectrum $s_d\big(H(P)\big)$ of the operator $H(P)$ was not studied and we knew nothing on the spectrum $s(H)$ of the operator $H$. In this talk we prove the following assertions:

1. For any positive atomic ions discrete spectrum $s_d\big(H(P)\big)$ is infinite.
2. Discrete spectrum $s_d\big(H(P)\big)$ may be infinite even for the systems $Z_1$ with short-range interactions between the particles.
3. The spectrum $s\big(H(P)\big)$ of the operator $H(P)$ does not depend on $P$
4. The spectrum $(H)$ coincides with the spectrum $s\big(H(P)\big)$, but $s(H)$ has an additional infinite degeneration in comparing with $s\big(H(P)\big)$; so for any positive atomic ions $s(H)$ consists of the infinite number of infinitely degenerated isolated eigenvalues, which go to some point $b$ and of the all points of half-line $[b, +\infty)$

**Remark**. Assertion 2 is not connected with Efimov effect. It is based on the infinite degeneration of lower bound of $s_e\big(H(P)\big)$ as the eigenvalue of the hamiltonian $H(Z_2; P)$, corresponding to some decomposition $Z_2$ of $Z_1$ into 2 non-interacting clusters.