AMP 2024
Analysis and Mathematical Physics
Book of Abstracts

Analysis and Mathematical Physics https://www.iimas.unam.mx/amp2024/ is an online conference aiming to bring together leading experts and young researchers from all over the world who work or are interested in mathematical problems within the context of mathematical physics. AMP conference's purpose is also to facilitate the exchange of ideas and help develop existing and future scientific collaborations. There will be several lectures on the topics; there will be no charge to speakers or participants, but registration is required. AMP conference will be an online event from August 5-17, 2024, on a ZOOM session, broadcast live on Facebook and YouTube IIMAS-UNAM's institutional channels.

Scope

The conference will concentrate on the following topics in mathematical analysis within the context of mathematical physics: Direct and inverse spectral and scattering theory for differential and difference equations and for systems of such equations. Differential operators on spatial networks. Differential operators on closed sets. Inverse problems for nonlocal operators. Orthogonal polynomials, Jacobi and CMV matrices. Quantum graphs. Applications of spectral and scattering theory to quantum mechanics and plasma physics.

Invited Speakers

Tuncay Aktosun (University of Texas at Arlington, USA)
Sergei Avdonin (University of Alaska Fairbanks, USA)
Jussi Behrndt (Graz University of Technology, Austria)
Gregory Berkolaiko (Texas A&M University, USA)
Natalia Bondarenko (Samara University, Russia)
Sergey Buterin (Saratov State University, Russia)

Abdon Choque-Rivero (Universidad Michoacana de San Nicolsá de Hidalgo, Mexico)

Jan Dereziński (University of Warsaw, Poland)

Bruno Després (Sorbonne Université, France)

Ramazan Ercan (California State University San Marcos, USA)

Pavel Exner (Nuclear Physics Institute - Academy of Sciences, Czechia)

Fritz Gesztesy (Baylor University, USA)

Mikhail Ignatiev (Saratov State University, Russia)

Evgeny Korotyaev (St. Petersburg State University, Russia)

Vladislav Kravchenko (Centro de Investigación y de Estudios Avanzados - Instituto Politécnico Nacional, Mexico)

Pavel Kurasov (Stockholm University, Sweden)

Maria Kuznetsova (Saratov State University, Russia)

Wencai Liu (Texas A&M University, USA)

Delio Mugnolo (FernUniversität in Hagen, Germany)

Vassilis Papanicolaou (National Technical University of Athens, Greece)

Olaf Post (University of Trier, Germany)

Julien Ricaud (Universidad Nacional Autónoma de México, Mexico)

Victor Rykhlov (Saratov State University, Russia)

Mostafa Sabri (NYU Abu Dhabi, Abu Dhabi)

Christiane Tretter (University of Bern, Switzerland)

Mehmet Unlu (Recep Tayyip Erdogan University, Turkey)

Ricardo Weder (Universidad Nacional Autónoma de México, Mexico)

Vjacheslav Yurko (Saratov State University, Russia)

Organizing Committee

Tuncay Aktosun (University of Texas at Arlington, USA)

Sergei Avdonin (University of Alaska Fairbanks, USA)

Ricardo Weder (Universidad Nacional Autónoma de México, Mexico)

Vjacheslav Yurko (Saratov State University, Russia)

Abstracts

Soliton solutions to a system of nonlinear evolution equations associated with a third-order ordinary linear differential equation

Tuncay Aktosun

University of Texas at Arlington, USA

Abstract

Using the recently developed solution method for the inverse scattering for the third-order linear equation

$$\frac{d^3\psi}{dx^3} + Q(x)\,\frac{d\psi}{dx} + P(x)\,\psi = k^3\,\psi, \qquad x \in \mathbb{R},$$

we obtain N-soliton solutions for the associated integrable nonlinear system of partial differential equations given by

$$\begin{cases} Q_t + Q_{xxxxx} + 5\,Q\,Q_{xxx} + 5\,Q_x\,Q_{xx} + 5\,Q^2\,Q_x + 15\,P\,Q_{xx} - 30\,P\,P_x + 15\,P_x\,Q_x = 0, \\ P_t + P_{xxxxx} + 5\,Q\,P_{xxx} + 15\,Q_x\,P_{xx} + 20\,P_x\,Q_{xx} + 5\,Q^2\,P_x + 10\,P\,Q_{xxx} \\ \qquad\qquad - 15\,P\,P_{xx} + 10\,P\,Q\,Q_x - 15\,(P_x)^2 = 0, \end{cases}$$

where Q and P are real-valued functions of the independent variables x and t and vanishing as $x \to \pm\infty$. An N-soliton solution for the integrable system under consideration contains $2N$ complex parameters that correspond to N bound-state energies and N time-evolved dependency constants for the associated linear system. The $2N$ complex parameters in this case is equivalent to $3N$ real parameters. In the special cases $P(x,t) \equiv 0$ or $P(x,t) \equiv Q_x(x,t)$, the integrable nonlinear system reduces to the integrable nonlinear evolution equation

$$Q_t + Q_{xxxxx} + 5\,Q\,Q_{xxx} + 5\,Q_x\,Q_{xx} + 5\,Q^2\,Q_x = 0,$$

which is known as the Sawada–Kotera equation or the Kaup–Kupershmidt equation, respectively. By using the appropriate restrictions on the dependency constants, we obtain N soliton solutions to the reduced integrable evolution equation, where an N-soliton solution is expressed in terms of $2N$ real parameters. This is joint work with Abdon Choque-Rivero, Ivan Toledo, and Mehmet Unlu, based on the solution method for the inverse scattering for the third-order linear equation developed in the PhD thesis of Ivan Toledo under the supervision of the presenter.

Inverse problems for the Schrödinger operator on metric graphs

Sergei Avdonin

University of Alaska Fairbanks, USA

Abstract

Inverse spectral theory for the Schrödinger operator on intervals (finite and infinite) of the real axis has been developed in classical papers by Borg, Levinson, Gelfand, Levitan, Krein, Marchenko. The theory of dynamical inverse problems was developed starting '70-ies. Along with important applications, this theory provides more simple proofs of many key results in inverse spectral theory. One of the achievements of the dynamical inverse theory is the boundary control method, which is based on deep connections of inverse problems with control and system theory, functional analysis and operator theory. Recent interest in inverse theory for operators on metric graphs is motivated by applications to important problems of classical and quantum physics, chemistry, biology, and engineering. For trees, i.e. graphs without cycles, various types of inverse problems were studied in the literature, but very little was done for graphs with cycles. In this talk we describe identifiability results for the Schrödinger operator on general compact graphs. We prove that the properly defined Titchmarsh–Weyl matrix function (or other equivalent spectral data) uniquely determines the potential, lengths of the edges, and the topology of the graph. We develop algorithms recovering the unknown parameters of the system and establish connections between the minimal number of observers and maximal multiplicity of the spectrum of the Schrödinger operator on the graph. Our approach combines the spectral and dynamical approaches including a new version of the recently developed leaf peeling method. It uses connections between controllability and identifiability of dynamical systems. In particular, we prove the exact controllability of the corresponding Schrödinger and wave equations on general graphs — the results presenting independent interest in control theory.

Spectral shift functions and Dirichlet-to-Neumann maps

Jussi Behrndt

Graz University of Technology, Austria

Abstract

The spectral shift function of a pair of self-adjoint operators is expressed via an abstract operator-valued Titchmarsh-Weyl m-function. This general result is applied to different self-adjoint realizations of second-order elliptic partial differential operators on smooth domains with compact boundaries, Schrödinger operators with compactly supported potentials and Schrödinger operators with singular potentials. In these applications the spectral shift function is determined in an explicit form with the help of (energy parameter dependent)

Dirichlet-to-Neumann maps. The talk is based on joint works with Fritz Gesztesy (Baylor University, Waco) and Shu Nakamura (Gakushuin University, Tokyo).

Morse theory for eigenvalues of self-adjoint families

Gregory Berkolaiko

Texas A&M University, USA

Abstract

The question of optimizing an eigenvalue of a family of self-adjoint operators that depends on a set of parameters arises in diverse areas of mathematical physics. Among the particular motivations for this talk are the Floquet-Bloch decomposition of the Schrödinger operator on a periodic structure, nodal count statistics of eigenfunctions of quantum graphs, conical points in potential energy surfaces in quantum chemistry and the minimal spectral partitions of domains. In each of these problems one seeks to identify and/or count the critical points of the eigenvalue with a given label (say, the third lowest) over the parameter space which is often known and simple, such as a torus. Classical Morse theory is a set of tools connecting the number of critical points of a smooth function on a manifold to the topological invariants of this manifold. However, the eigenvalues are not smooth due to presence of eigenvalue multiplicities or "diabolical points". We rectify this problem for eigenvalues of generic families of finite-dimensional operators. The "diabolical contribution" to the "Morse indices" of the problematic points turns out to be universal: it depends only on the multiplicity and the relative position of the eigenvalue of interest and not on the particulars of the operator family. Using tools such as Clarke subdifferential and stratified Morse theory of Goresky–MacPherson, we express the "diabolical contribution" in terms of homology of Grassmannians of appropriate dimensions. Based on a joint work with Igor Zelenko (TAMU).

Inverse problems for the fourth-order differential operators

Natalia Bondarenko

Samara University, Russia

Abstract

In this talk, we will consider two inverse spectral problems for the fourth-order differential operators. The first problem was introduced by Barcilon in 1974 and consists in the recovery

of the differential operator coefficients from three spectra. The second problem was introduced by McLaughlin in 1982 and consists in the recovery of the operator from the spectrum and the two sets of norming constants. Although these problems started to be investigated in 1970-1980s, to the best of the author's knowledge, there were no rigorous uniqueness results for them. In this talk, we discuss the question of uniqueness for Barcilon's and McLaughlin's problems by using the general inverse problem theory for higher-order differential operators created by Yurko.

On damping a control system of arbitrary order with global aftereffect on a temporal tree

Sergey Buterin

Saratov State University, Russia

Abstract

The so-called quantum graphs model various processes in complex systems represented as spatial networks in many fields of science and technology. An example is elastic string networks, at whose nodes, in addition to the continuity conditions, there appear also Kirchhoff conditions, which express a balance of tensions. However, we suggest a different look at quantum graphs as temporal networks, when the variable parameterizing the edges is identified with time. Then each internal vertex is understood as a moment in time giving several different scenarios for the further flow of a process. Under such settings, we study the problem of damping a control system described by functional-differential equations of arbitrary order and neutral type with non-smooth complex coefficients on an arbitrary tree with global delay. The latter means that the delay propagates through all internal vertices of the tree. It is noteworthy that Kirchhoff-type conditions emerge here too. Namely, they will be satisfied by such a trajectory of the process flow that is optimal with account of all possible scenarios simultaneously. Its existence and uniqueness are established. Previously, such problems were studied only on an interval for control systems of the first order with constant or smooth coefficients. The non-smooth case has required introducing a family of special nonlocal quasi-derivatives. In the talk, it is also planned to compare them with the classical quasi-derivatives for ordinary differential operators.

The matrix Toda equations

Abdon Choque-Rivero

Universidad Michoacana de San Nicolás de Hidalgo, Mexico

Abstract

Let σ be a monotone nondecreasing $q \times q$ matrix function $\sigma : [0, +\infty) \to \mathbb{R}$ satisfying $\sigma(x+0) = \sigma(x)$ for $x \in [0, +\infty)$. For $\alpha \geq 0$, we define the matrix moments $s_j(\alpha)$ by using the Laplace transform

$$s_j(\alpha) := \int_0^\infty x^j e^{-\alpha x} d\sigma(x), \quad j \in \mathbb{N} \cup \{0\},$$

and we construct [1] a family of orthogonal matrix polynomials $\{P_j(x,\alpha)\}_{j\geq 0}$ with respect to $e^{-\alpha x} d\sigma(x)$. We also construct the corresponding family of second kind polynomials $\{Q_j(x,\alpha)\}_{j\geq 0}$. We derive the matrix Toda equations associated with the matrix coefficients $A_j(\alpha)$ and $B_j(\alpha)$ appearing in the discrete linear system

$$\begin{aligned} P_{j+1}(x,\alpha) =& (xI_q - A_j(\alpha))P_j(x,\alpha) - B_{j-1}(\alpha)^* P_{j-1}(x,\alpha), \quad j \geq 1, \\ P_0(x,\alpha) =& I_q, \quad P_1(x,\alpha) = xI_q - A_0(\alpha), \end{aligned}$$

for $\alpha \in [0, +\infty)$, where I_q is the $q \times q$ unit matrix and the asterisk denotes the matrix adjoint. Additionally, we introduce [2] a certain family of Hurwitz matrix polynomials $f_j(x,\alpha)$ for $j \geq 0$ associated with the matrix polynomials $P_j(x,\alpha)$ and $Q_j(x,\alpha)$T

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Propagators on curved spacetimes

Jan Dereziński

University of Warsaw, Poland

Abstract

Operator theory is very useful in Quantum Field Theory on curved spacetimes. On a large class of globally hyperbolic spacetimes one can define four natural Green functions of the Klein-Gordon equation. As is well-known, we have the forward propagator and the backward

propagator. It is less known that usually one can also define the (distinguished) Feynman and anti-Feynman propagator, used in the calculation of Feynman diagrams. I will discuss how to define these objects in various kinds of spacetimes: e.g. the Friedman-Lemaitre-Robinson-Walker spacetime, the DeSitter and Anti-deSitter space.

Scattering structure of linearized Vlasov-Poisson equations

Bruno Després

Sorbonne Université, France

Abstract

It has recently been showed that scattering theory (à la Kato-Lax) can be used to reformulate linear Vlasov-Poisson equation in the form of a Vlasov-Ampere system. It yields a simple explanation of linear Landau damping. I will show various extensions to other problems derived from particles physics. These works have been obtained with F. Charles, R. Weder, A. Rege, C. Buet and V. Fournet.

The direct and inverse scattering problems for the first-order linear discrete system associated with the derivative NLS system

Ramazan Ercan

California State University San Marcos, USA

Abstract

The direct and inverse scattering problems are analyzed for a first-order linear discrete system associated with the integrable semi-discrete version of the derivative NLS (nonlinear Schrödinger) system. The Jost solutions, the scattering coefficients, and the bound-state dependency and norming constants are investigated and related to the corresponding quantities for two particular discrete linear systems associated with the integrable semi-discrete version of the NLS system. The bound-state data set with any number of bound states and with any multiplicities is described in an elegant manner in terms of a pair of constant matrix triplets. Several methods are presented to solve the inverse problem for the corresponding linear system. One of these methods involves a discrete Marchenko system using as input the scattering data set consisting of the scattering coefficients and the bound-state information, and this method is presented in a way that is generalizable to other first-order systems both in the discrete and continuous cases for which a Marchenko system is not yet available.

Schrödinger operators with channel-type potentials

Pavel Exner

Doppler Institute for Mathematical Physics and Applied Mathematics, Czechia

Abstract

We examine Schödinger operators for which the potential is a channel of a fixed profile, with the focus on relations between the spectrum and the channel geometry. We provide sufficient conditions under which a nonstraight but asymptotically straight channel gives rise to a non-empty discrete spectrum. We also address the ground-state optimalisation in case of a loop-shaped configuration, and mention the situation then the channel is replaced by an array of potential wells.

Continuity properties of the spectral shift function for massless Dirac operators and an application to the Witten index

Fritz Gesztesy

Baylor University, USA

Abstract

We report on results regarding the limiting absorption principle for multi-dimensional, massless Dirac-type operators (implying absence of singularly continuous spectrum) and continuity properties of the associated spectral shift function.We will motivate our interest in this circle of ideas by briefly describing the connection to the notion of the Witten index for a certain class of non-Fredholm operators.This is based on various joint work with A. Carey, J. Kaad, G. Levitina, R. Nichols, D. Potapov, F. Sukochev, and D. Zanin.

Direct and inverse scattering for differential systems with a singularity

Mikhail Ignatiev

Saratov State University, Russia

Abstract

Scattering problem for the differential system: $y' = (\rho B + x^{-1}A + q(x))\, y$ on the semi-axis $x > 0$ is studied. Here ρ is the spectral parameter, $A, B, q(x)$ are $n \times n$, where $n > 2$, matrices, A, B are constant, $B = \mathrm{diag}(b_1, \ldots, b_n)$. Most general case of noncollinear $\{b_k\}$ and $q(\cdot) \in L_1(0, \infty) \cap L_p(0, \infty)$ is considered. We discuss direct scattering problem including construction and properties of certain distinguished fundamental system of solutions and inverse

scattering problem including constructive procedure of its solution and characterization of scattering data.

Schr̈odinger operators on periodic discrete and metric graphs

Evgeny Korotyaev

St. Petersburg State University, Russia

Abstract

Firstly we discuss the Schrödinger operator $H = -\Delta + V$ on the periodic discrete graph G, where the potential V is real and $V(x) \to 0$ as $|x| \to \infty$. We describe different spectral properties of H. Moreover, for the case of the cubic lattice we solve the inverse problem. Secondly, we discuss Schrödinger operators $\mathcal{H} = -\Delta_m + \mathcal{V}$ on the periodic metric graph $\mathcal{G}$ where the potential $\mathcal{V}$ is real and $\mathcal{V}(x) \to 0$ as $|x| \to \infty$. We describe different spectral properties of $\mathcal{H}$. These results are obtained jointly with Isozaki, Moller, Rasmussen, Saburova

Analytic representations for solutions in inverse coefficient problems

Vladislav Kravchenko

Centro de Investigación y de Estudios Avanzados - Instituto Politécnico Nacional, Mexico

Abstract

A unified approach to solving a variety of inverse one-dimensional coefficient problems is the subject of this talk. It is based on the recently discovered Neumann series of Bessel functions representations for solutions of the Sturm-Liouville equation [1], which are convenient for solving direct and inverse problems on a finite interval, as well as on the special power series representations for Jost solutions [2], well suited for direct and inverse problems on infinite intervals (see the monographs [3], [4] and references therein). The series representations possess remarkable convergence properties, such as the uniform convergence with respect to $\sqrt{\lambda} \in \mathbb{R}$, where λ is the spectral parameter of the equation. The knowledge of the first coefficient of any such series representation is sufficient for recovering the Sturm-Liouville equation. These features make the representations especially convenient for solving inverse coefficient problems. Difficult and numerically challenging inverse problems are reduced to systems of linear algebraic equations for the coefficients of the representations, and the Sturm-Liouville equation is recovered from the first coefficient. In particular, we discuss a

general inverse coefficient problem for a Sturm-Liouville equation with an unknown complex valued coefficient [5]. Special cases of the problem include the inverse two-spectrum Sturm-Liouville problem (see, e.g., [6], [7]), the recovery of the potential from a Weyl function [7], [8], [9], [10], [11], the inverse scattering problem for a finitely supported potential [12] and the inverse transmission eigenvalues problem (see, e.g., [13], [14], [15]) among others. A variety of inverse coefficient problems for partial differential equations and for quantum graphs are also reduced to the considered problem. The approach leads to a simple and efficient numerical algorithm, that is illustrated by numerical examples.The classical direct and inverse scattering problems on the line are also discussed. The special power series representation for Jost solutions lends itself to an efficient numerical approach to solving both problems that leads to a realization of the inverse scattering transform method for solving nonlinear partial differential equations [16].

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[16] S. M. Grudsky, V. V. Kravchenko, S. M. Torba Realization of the inverse scattering transform method for the Korteweg-de Vries equation. *Math. Methods Appl. Sci.* 46 (2023), 9217-9251.

Maximal dissipative operators on metric graphs: hypergraphs and multiplicities

Pavel Kurasov

Stockholm University, Sweden

Abstract

Dissipative Schrödinger operators on metric graphs will be discussed. Vertex conditions and potentials leading to maximal dissipative operators are characterised. The language of hypergraphs is introduced and used to determine possible multiplicity of the eigenvalues of self-adjoint reductions, which depends not only on the properties of the potential but on the topologic and geometric properties of the metric graph. This leads to characterisation of all operators not possessing any self-adjoint reduction, so-called completely non-self-adjoint

operators, on compact metric graphs with delta couplings at the vertices. This is joint work with Jacob Muller (Stockholm) and Sergey Naboko (St Petersburg).

On inverse problems for operators with frozen argument

Maria Kuznetsova

Saratov State University, Russia

Abstract

The talk is devoted to Sturm-Liouville operator perturbed by the member with frozen argument. This member contains the value of the input function at a fixed point multiplied by a potential. We study the problem of recovering the potential from the spectrum of the operator and from necessary additional data. For the inverse problem, we obtain characterization of the input data, a uniqueness theorem, and global stability. The talk is partly based on the results of the following papers:

[1] Kuznetsova, M. On recovering non-local perturbation of non-selfadjoint Sturm-Liouville operator. Izvestiya of Saratov University. Mathematics. Mechanics. Informatics (to appear). https://doi.org/10.48550/arXiv.2307.10075

[2] Kuznetsova, M. Uniform stability of recovering Sturm-Liouville-type operators with frozen argument. Results Math 78, 169 (2023). https://doi.org/10.1007/s00025-023-01945-z

Algebraic geometry, complex analysis and combinatorics in spectral theory of periodic graph operators

Wencai Liu

Texas A&M University, USA

Abstract

In this talk, we will discuss the significant role that the algebraic and analytic properties of complex Bloch and Fermi varieties play in the study of periodic operators. I will begin by highlighting recent discoveries about these properties, especially their irreducibility. Then, I will show how we can use these findings, together with techniques from complex analysis and combinatorics, to study spectral and inverse spectral problems arising from periodic operators.

The heat kernel on metric graphs and its geometric significance

Delio Mugnolo

FernUniversität in Hagen, Germany

Abstract

The heat semigroup on a compact metric graph $\mathcal{G}$ is known to consist of integral operators: its kernel $p = p_t(x, y)$ is known as the *heat kernel* of $\mathcal{G}$. It has been known since the 1980s that the *heat trace* – i.e., the integral of p_t along the diagonal – reveals significant geometric and topologic information in the limit $t \to 0$. We are going to investigate further integrated versions of the heat kernel and delve into recently discovered geometric aspects of their theory. This is based on joint work with Patrizio Bifulco, Marvin Plümer and Matthias Täufer (Hagen).

Discrete and continuous non-self-adjoint Hill operators whose spectrum is a real interval

Vassilis Papanicolaou

National Technical University of Athens, Greece

Abstract

We derive certain properties of the general discrete second-order periodic operator on the integer lattice with complex coefficients. In particular, we investigate the case where the spectrum is an interval of the real line. Recall that in the discrete case there is no Liouville transformation which transforms the general second-order operator to a (discrete) Schrödinger operator. We also we discuss a conjecture regarding the continuous Hill operator with a complex potential whose spectrum is the positive real axis. Such potentials have some physical significance (PT symmetry).

Different variants of generalised operator norm convergence

Olaf Post

University of Trier, Germany

Abstract

In this talk, we present different concepts of how to compare the distance in operator norm of operators acting in different Hilbert spaces. We establish equivalence of the distances in certain situations and compare it with some spectral distance. We also introduce a related notion of convergence, applicable for resolvents of operators acting in varying Hilbert spaces. The talk is based on joint work with Sebastian Zimmer, Uni Trier.

Spectral stability in the nonlinear Dirac equation with Soler-type nonlinearity

Julien Ricaud

Universidad Nacional Autónoma de México, Mexico

Abstract

This talk concerns the (generalized) Soler model: a nonlinear (massive) Dirac equation with a nonlinearity taking the form of a space-dependent mass. The equation admits standing wave solutions and they are generally expected to be stable (i.e., small perturbations in the initial conditions stay small) based on numerical simulations. However, contrarily to the nonlinear Schrödinger equation for example, there are very few results in this direction. The results that I will discuss concern the simpler question of spectral stability (and instability), i.e., the absence (or presence) of exponentially growing solutions to the linearized equation around a solitary wave. As in the case of the nonlinear Schrödinger equation, this is equivalent to the presence or absence of "unstable eigenvalues" of a non-selfadjoint operator with a particular block structure. I will highlight the differences and similarities with the Schrödinger case, present results for the one-dimensional case, and discuss open problems. This is joint work with Danko Aldunate, Edgardo Stockmeyer, and Hanne Van Den Bosch.

On multiple completeness of root functions of certain classes of ordinary differential pencils of operators

Victor Rykhlov

Saratov State University, Russia

Abstract

In the space of square-integrable functions on a finite interval, we consider the class of polynomial ordinary differential pencils of operators generated by a homogeneous differential expression of n-th order with constant coefficients and homogeneous two point (at the ends of the main interval) boundary conditions. It is assumed that the roots of the characteristic equation of this class of pencils are simple and nonzero. In the space of square-integrable functions on the interval under consideration, the m-fold completeness ($1 \leq m \leq n$) of the system of root functions of the pencils from this class is investigated. To study multiple completeness, a method of generalized generating functions (for the system of root functions) depending on a vector parameter is proposed. The essence of the method is to construct such vector parameters for which the classical scheme for proving multiple completeness of root

functions is preserved. Sufficient conditions for n- and m-fold completeness of the system of root functions of the pencils from this class are formulated. Applications of these sufficient conditions to certain classes of differential pencils of operators are considered.

Ergodic Theorems for Continuous-time Quantum Walks on Crystal Lattices and the Torus

Mostafa Sabri

NYU Abu Dhabi, Abu Dhabi

Abstract

Abstract: We give several quantum dynamical analogs of the classical Kronecker-Weyl theorem, which says that the trajectory of free motion on the torus along almost every direction tends to equidistribute. As a quantum analog, we study the quantum walk exp(?it?)? starting from a localized initial state ?. Then the flow will be ergodic if this evolved state becomes equidistributed as time goes on. We prove that this is indeed the case for evolutions on the flat torus, provided we start from a point mass, and we prove discrete analogs of this result for crystal lattices. On some periodic graphs, the mass spreads out non-uniformly, on others it stays localized. Finally, we give examples of quantum evolutions on the sphere which do not equidistribute. Joint work with Anne Boutet de Monvel.

Challenges for non-selfadjoint spectral problems in analysis and computation

Christiane Tretter

University of Bern, Switzerland

Abstract

Non-selfadjoint spectral problems appear frequently in a wide range of applications. Reliable information about their spectra is therefore crucial, yet extremely difficult to obtain by approximations. This talk focuses on tools to master these challenges such as spectral pollution or spectral invisibility. In particular, the concept of essential numerical range for unbounded linear operators is introduced and studied, including possible equivalent characterizations and perturbation results. Compared to the bounded case, new interesting phenomena arise which are illustrated by some striking examples. A key feature of the essential numerical range is that it captures, in a unified and minimal way, spectral pollution which may affect e.g. spectral approximations of PDEs by projection methods or domain truncation methods.

As an application, Maxwell's equations with conductivity will be considered. Joint work with S. Boegli, M. Marletta, and F. Ferraresso.

The Marchenko method for the general system of derivative nonlinear Schrödinger Equations

Mehmet Unlu

Recep Tayyip Erdogan University, Turkey

Abstract

A system of Marchenko integral equations is presented to solve the inverse scattering problem for the linear system related to the general system of derivative nonlinear Schrödinger equations. The recovery of the two potentials and the Jost solutions from the solution to the Marchenko system is outlined. Some explicit solution formulas are provided in the reflectionless case. In the reduced case, i.e. when the two potentials in the linear system are related to each other through complex conjugation, the corresponding reduced Marchenko integral equation is obtained. The theory presented is illustrated with some explicit examples.

The Bernstein-Landau paradox in fusion plasmas An operator theory point of view

Ricardo Weder

Universidad Nacional Autónoma de México, Mexico

Abstract

The Landau damping is a fundamental phenomenon in plasma physics. It was discovered by L. Landau, and it can be briefly described as the decay in time, the damping, of the electric field in the collisionless motion of a plasma. It is a striking fact that this decay, that can even be exponential, appears in the absence of any dissipation. There is a large mathematics and physics literature in Landau damping, and currently it is intensively studied in the mathematics literature, both in the linear and the nonlinear cases. The Landau damping appears in the absence of a magnetic field. It is a remarkable fact that for magnetized plasmas the behavior of the electric field changes drastically and has an oscillatory behaviour in time, no matter how small the magnetic field is. This fundamental fact was discovered by I. B. Bernstein and it is known as the Landau-Bernstein paradox, because it seems paradoxical that even an arbitrary small, but nonzero, value of the external constant magnetic field can

be the cause of this radical change in the behaviour of the electric field for large times. In this talk we revisit the Bernstein-Landau paradox and we study it from a new point of view. Instead of considering the Vlasov-Poisson equations, as it is usually done in the literature, we reformulate the problem in terms of the Vlasov-Ampère equations. This allows us to state the problem as a Schrödinger equation in the Hilbert space of states of finite energy, where the selfadjoint Hamiltonian is replaced by the Vlasov-Ampère operator that we introduce. In this way we explain the Bernstein-Landau paradox as a problem in singular perturbation theory, where the spectrum of the Vlasov-Ampère operator suddendly changes from pure point to absolutely continuos. We prove this in two different ways, by the explicit calculation of a complete set of eigenfunctions of the Vlasov-Ampère operator, and by an operator theoretical argument. We also present numerical results that illustrate the Landau damping and the Bernstein-Landau paradox. This talk is based in joint work with Frédérique Charles, Bruno Després, and Alexandre Rege [1].

[1] Frédérique Charles, Bruno Després, and Alexandre Rege , The magnetized Vlasov-Ampère system and the Bernstein-Landau paradox, J. Stat. Phys. **183** 23 (2021).

Method of spectral mappings in the inverse problem theory

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Abstract

Inverse spectral problems will be considered for ordinary differential equations. We pay the main attention on the method of spectral mappings which is an effective tool for studying this class of inverse problems. We give a short review on results obtained by this method, namely, results on inverse spectral problems for arbitrary order of differential operators, for canonical systems, for differential operator on spatial networks and others.