

# COLOQUIO DE ANÁLISIS Y FÍSICA–MATEMÁTICA

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## EXISTENCIA CASI–GLOBAL DE SOLUCIONES A ECUACIONES DE KLEIN–GORDON HAMILTONIANAS, SEMI–LINEALES, CON DATOS DE CAUCHY PEQUEÑOS EN VARIEDADES DE ZOLL

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### Resumen

En este coloquio se discutirá la existencia casi global de soluciones a ecuaciones de Klein–Gordon en variedades de Zoll (por ejemplo esferas de dimensión arbitraria). Las demostraciones están basadas en métodos de forma normal de Birkhoff y en la distribución de los autovalores del Laplaciano perturbado por un potencial en variedades de Zoll.

14 de diciembre de 2005.



## INTRODUCTION TO CALDERÓN'S PROBLEM AND OUTLINE OF THE NEW COMPLEX ANALYSIS METHOD

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### Abstract

The basic problem of *Electrical Impedance Tomography* is to determine the conductivity of a body by measuring voltages and currents on its surface. It has been proposed as a valuable diagnostic tool especially for detecting breast cancer and for monitoring heart and lungs.

Is this really possible? Can one formulate the problem in mathematical terms?

The answers to both questions are positive! In 1980 A.P. Calderón put the question in to the language of mathematics by formulating it in the following way:

Suppose  $\Omega \subset \mathbb{R}^n$  is smooth and bounded,  $\sigma \in L^\infty(\Omega)$  is bounded away from zero, and that  $u \in H^1(\Omega)$  is the weak solution of

$$\operatorname{div}(\sigma \nabla u) = 0$$

$$u|_{\partial\Omega} = f$$

where  $f \in H^{1/2}(\partial\Omega)$ . Define the Dirichlet–to–Neumann map:  $\Lambda_\sigma H^{1/2}(\partial\Omega) \rightarrow H^{-1/2}(\partial\Omega)$  by

$$\Lambda_\sigma(f) = \sigma \frac{\partial u}{\partial \nu} \Big|_{\partial\Omega}$$

where  $\nu$  is the unit outer normal to  $\partial\Omega$ . Does  $\Lambda_\sigma$  uniquely determine  $\sigma$ ? In physical terms  $u|_{\partial\Omega} = f$  represents the electric potential i.e. voltage and  $\sigma \frac{\partial u}{\partial \nu}|_{\partial\Omega}$  the electric current at the boundary of  $\Omega$ .

In the four lectures we will discuss the history of this problem and introduce some new techniques of complex analysis to attack the problem. We demonstrate how this problem can completely and constructively be solved in two dimensions in its original form. In particular, we prove uniqueness of the inverse problem. The methods involve quasiconformal techniques, harmonic analysis, Beals–Coifman theory and finally some topological arguments. The work is a joint study with K. Astala from Helsinki.

*16 de noviembre de 2005.*



### **GEL'FAND INVERSE BOUNDARY SPECTRAL PROBLEM**

- ✦ Gel'fand inverse boundary spectral problem for a Schrödinger operator on a Riemannian manifold: Analytical and geometrical aspects of the boundary control method. Part I.
- ✦ Gel'fand inverse boundary spectral problem for a Schrödinger operator on a Riemannian manifold: Analytical and geometrical aspects of the boundary control method Part II.
- ✦ Inverse boundary value problem for the Dirac and Maxwell equations on a manifold.
- ✦ Geometric convergence and stability in the Gel'fand inverse problem.

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*20, 21, 26 y 28 de abril de 2005.*