COLOQUIO DE ANÁLISIS Y FÍSICA–MATEMÁTICA

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SPECTRAL THEORY OF QUANTUM GRAPHS

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Abstract

Under quantum graphs we understand differential operators on metric graphs. Such operators are determined by: 1) a metric graph; 2) differential expressions on the edges and 3) boundary conditions at the vertices.

These operators form a special class prossessing features of both ordinary and partial differential operators. Quantum graphs can be used to model narrow wave guides, quantum wires, carbon nano–structures, etcetera, and serve as toy models in the theory of dynamical systems and quantum mechanics. The spectral theory of quantum graphs is a rapidly developing area of research in modern mathematical physics. This lecture course is devoted to modern developments in the area and will be suitable as an introduction into this research area.

Three types of quantum graphs will attract our attention:

- compact finite graphs;
- noncompact graphs obtained by attaching few infinite edges;
- self–similar graphs.

It has been shown that the spectrum of finite compact graphs is pure discrete and in order to calculate it one way view these graphs as quantum billiards. In this way it is possible to establish a trace formula connecting the spectrum of a quantum graph and the set of periodic orbits of the underlying metric graph. Moreover, in this case the inverse spectral problem will be discussed.

The spectrum of non–compact graphs contain branches of absolutely continuous spectrum which gives rise to the study of direct and inverse scattering problems. Such models are important for the calculation of transition probabilities in quantum networks.

Self–similar graphs serve as models of fractals and attract attention due to unusual spectral properties.

The preliminary plan for the course looks as follows:

1. Quantum graphs as coupled ordinary differential operators: definition, self–adjoint vertex conditions, elementary spectral properties and counterexamples.
2. Trace formula for graph Laplacians: spectral and topological invariants. Can one hear the shape of a graph?
3. On inverse scattering problems for noncompact graphs: uniqueness theorems and explanation of counterexamples.
4. Graphs with rationally dependent lengths of edges and self–similar graphs (fractal models).

13, 15, 20 y 22 de marzo de 2007.

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PROBABILIDADES CONDICIONALES Y COLAPSO EN LAS MEDICIONES CUÁNTICAS

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Resumen

Incluyendo el sistema y el instrumento de medición en la descripción cuántica del proceso de medición, y usando el concepto de probabilidades condicionales, es posible deducir el operador estadístico del sistema después de la medición. Este operador estadístico determina la distribución de probabilidad para todas las mediciones posteriores, y en general no coincide con el que se obtiene usando el postulado del colapso.

23 de enero de 2007.