COLOQUIO DE ANÁLISIS Y FÍSICA–MATEMÁTICA

Organizers: Dr. Rafael del Río y Dr. Ricardo Weder

RIEMANN–HILBERT PROBLEM APPROACH TO LONG–TIME ASYMPTOTICS FOR NONLINEAR INTEGRABLE PROBLEMS, II: THE CAMASSA–HOLM EQUATION

Dra. Anne Boutet de Monvel
Universidad de París

Abstract

We develop a Riemann–Hilbert approach for the Camassa–Holm equation $u_t - u_{txx} + 2u_x + 3uu_x = 2u_x u_{xx} + uu_{xxx}$ on the line $-\infty < x < \infty$. This approach allows us to study the long–time behavior of solutions of initial value problems for this equation. Regions of the half plane $-\infty < x < \infty, t \geq 0,$ with different long–time behavior are specified.

We obtain analogous results for initial–boundary value problems for the Camassa–Holm equation on the half–line $x > 0$ with time decaying boundary conditions at $x = 0$. Regions of the quarter plane $x \geq 0, t \geq 0,$ with different long–time behavior are specified.

Work in collaboration with Dmitry Shepelsky.

27 de mayo de 2008.

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RIEMANN–HILBERT PROBLEM APPROACH TO LONG–TIME ASYMPTOTICS FOR NONLINEAR INTEGRABLE PROBLEMS, I: THE NONLINEAR SCHRÖDINGER EQUATION

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Abstract

We develop a Riemann–Hilbert approach to the focusing nonlinear Schrödinger equation $iq_t + q_{xx} + 2|q|^2 q = 0$, on the first quarter plane $x > 0, t > 0$ for fast decaying initial data $q_0(x)$ and time–periodic boundary data $g_0(t)$. This approach allows us to study the long–time behavior of solutions of this initial boundary value problem. Regions of the quarter plane $x \geq 0, t \geq 0$ with different long–time behavior are specified.

However, this approach requires certain a priori information about the long–time behavior of the boundary values which do not involved in the formulation of a well–posed initial boundary value problem (for example, the Neumann boundary values for the Dirichlet problem). Various possibilities for this are discussed and illustrated numerically in the case of one–frequency boundary data.

Work in collaboration with Alexander Its, Vladimir Kotlyarov, Dmitry Shepelsky, and Chunxiong Zheng (numerics).

20 de mayo de 2008.

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ON THE INVERSE SCATTERING PROBLEM FOR THE HELMHOLTZ EQUATION ON THE REAL LINE

Dra. Ingrid Beltită
Institute of Mathematics "Simion Stoilow" of the Romanian Academy

Abstract

We consider the inverse scattering problem for the equation

$$u'' + \frac{k^2}{c^2}u = 0,$$

on $\Re$, where c is a real measurable function satisfying $c(x) \geq c_m > 0$ and $\left| c(x) - 1 \right| \leq C\langle x \rangle^{-1-\delta}, \delta > 0.$ The situation is very different from the case of the Schrödinger operators, since the Jost functions and the scattering coefficients do not have simple behaviour at infinity in the energy variable. We analyze this behaviour and show that the analysis of analytic functions (but not necessarily of Hardy classes) on the half–spaces plays an important role in proving the uniqueness in the inverse scattering problem.

14 de mayo de 2008.

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LOCAL SMOOTHING FOR THE BACKSCATTERING TRANSFORM FOR SCHRÖDINGER OPERATORS

Dra. Ingrid Beltită
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Abstract

We consider the inverse backscattering problem for Schrödinger operators $H_v = -\Delta + v, v \in L^\infty_{comp}(\Re^n; \Re)$, in odd dimensions $n \geq 3$. The backscattering transform Bv of the potential v is, up to a smooth function, the real part of the inverse Fourier transform of the backscattering part of the scattering matrix. The mapping $L^\infty_{comp}(\Re^n) \ni v \mapsto Bv \in D'(\Re^n)$ is entire analytic, and we write

$$Bv = \sum_1^\infty B_N v,$$

where $B_N v$ is the $N : th$ order term in the power series expansion of B at $v = 0$. Here $B_1 v = v$. We shall present estimates for $B_N v$ in Sobolev spaces $H_{(s)}(\Re^n)$ and show that $v \mapsto Bv$ extends to an entire analytic mapping on $H_{(s)}(\Re^n) \cap \varepsilon'(\Re^n)$ with values in $H_{(s),comp}(\Re^n)$, when $s \geq (n-3)/2$. We show moreover that, when $s > (n-3)/2$ and $v \in H_{(s)}(\Re^n) \cap \varepsilon'(\Re^n)$, the regularity of $B_N v$ increases with N and $v - Bv$ is locally of class $H_{(s+a)}(\Re^n)$, where $0 \leq a < 1, a < s - (n-3)/2$.

This talk reports on joint work with Anders Melin (Lund University).

6 de mayo de 2008.

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RECUPERACIÓN DE LA FUNCIÓN DE GREEN EN ELASTICIDAD DINÁMICA A PARTIR DE CORRELACIONES DEL RUIDO

Dr. Francisco J. Sánchez–Sesma
Instituto de Ingeniería–UNAM

26 de febrero de 2008.