SEMINARIO DE OPERADORES Y FÍSICA–MATEMÁTICA

Organizers: Dr. Ricardo Weder y Dr. Rafael del Río

EXTENSIONS AND APPLICATIONS OF THE BEURLING–MALLIAVIN THEORY

Dr. Alexei Poltoratski
Texas A & M University

Abstract

Problems from several classical areas of analysis, such as completeness problems for sequences of reproducing kernels in Hilbert spaces of analytic functions, completeness problems for collections of special functions (Bessel, Airy, ...) in L2, spectral problems for Schrödinger and Krein string operators, etcetera, can be reduced to questions about triviality of kernels of Toeplitz operators. Adopting the "Toeplitz approach" allows one to put all these areas into one picture, which helps find the right questions and select the best tools to answer them. In my talk I will try to illustrate this approach by discussing new generalizations and applications of the so–called Beurling–Malliavin theory.

14 de diciembre de 2005.

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POTENTIALS PRESERVING ABSOLUTELY CONTINUOUS SPECTRUM – FROM POTENTIALS IN SOBOLEV SPACES WITH POSITIVE INDEXES TO THE ONES WITH NEGATIVE. SOME OPTIMAL RESULTS

Dr. Alexei Rybkin
University of Alaska Fairbanks

Abstract

We introduce a new series of trace type relations for 1D Schrödinger operators which may be interpreted as well–known Faddeev–Zhakharov (FZ) trace formulas at finite energies. Comparing to the original FZ formulas, these relations look quite unwieldy but have an important advantage – they can be properly closured for larger classes of distributional (singular) potentials. We discuss a general way to define the Schrödinger operator with potentials which are generalized functions (i.e. not necessarily locally summable). We then apply the new trace relations to prove the stability of the absolutely continuous spectrum and to derive new Lieb–Thirring type bounds for such potentials. Our results extend the relevant results due to Deift–Killip, Molchanov–Novitskii–Vainberg and others to much broader classes of potentials including delta, Coulomb, oscillatory to name just three. These classes can adequately be described in terms of proper Sobolev classes with negative indexes.

23 de noviembre de 2005.

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- **SOLUTIONS OF COMPLEX GEOMETRIC OPTICS**
- **μ–HILBERT TRANSFORM AND BOUNDARY INTEGRAL EQUATIONS**
- **TRANSPORT MATRIZ**

Dr. Lassi Päivärinta
Helsinki University

Abstract

The basic problem of *Electrical Impedance Tomography* is to determine the conductivity of a body by measuring voltages and currents on it's surface. It has been proposed as a valuable diagnostic tool especially for detecting breast cancer and for monitoring heart and lungs.

Is this really possible? Can one formulate the problem in mathematical terms?

The answers to both questions are positive! In 1980 A. P. Calderón put the question in to the language of mathematics by formulating it in the following way:

Suppose $\Omega \subset R^n$ is smooth and bounded, $\sigma \in L^\infty(\Omega)$ is bounded away from zero, and that $u \in H^1(\Omega)$ is the weak solution of

$$div(\sigma \nabla u) = 0$$
$$u|_{\partial\Omega} = f$$

where $f \in H^{1/2}(\partial\Omega)$. Define the Dirichlet –to– Neumann map: $\Lambda_\sigma H^{1/2}(\partial\Omega) \to H^{-1/2}(\partial\Omega)$ by

$$\Lambda_\sigma(f) = \sigma \frac{\partial u}{\partial v}|_{\partial\Omega}$$

where v is the unit outer normal to $\partial\Omega$. Does Λ_σ uniquely determine σ? In physical terms $u|_{\partial\Omega} = f$ represents the electric potential i.e. voltage and $\sigma \frac{\partial u}{\partial v}|_{\partial\Omega}$ the electric current at the boundary of Ω.

In the four lectures we will discuss the history of this problem and introduce some new techniques of complex analysis to attack the problem. We demonstrate how this problem can completely and constructively be solved in two dimensions in it's original form. In particular, we prove uniqueness of the inverse problem. The methods involve quasiconformal techniques, harmonic analysis, Beals–Coifman theory and finally some topological arguments. The work is a joint study with K. Astala from Helsinki.

17, 22 y 24 de noviembre de 2005.

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ON THE ACCUMULATION SPECTRUM OF SELF–ADJOINT OPERATORS

Dr. Andreas M. Hinz
Munich University

Abstract

We develop a couple of characterizations of the essential spectrum to obtain quantitative bounds for the distance of a real number from the essential spectrum of a self–adjoint operator in a Hilbert space.

The notion of accumulation spectrum is introduced as a natural extension to possibly infinite numbers. For the special case of Schrödinger operators in L_2 this leads to a better understanding of the infimum of the essential spectrum and of a method to investigate the behavior of eigensolutions at infinity.

3 de noviembre de 2005.

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SEMICIRCLE LAW FOR RANDOM MATRICES WITH CORRELATIONS

Dr. Hermann Schulz–Baldes
Erlangen University

Abstract

For a class of random matrix ensembles with correlated matrix elements, it is shown that the density of states is given by the Wigner semi–circle law. This is applied to effective Hamiltonians related to the Anderson model in dimensions greater than or equal to two.

29 de septiembre de 2005.

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DIFFUSION AND MIXING IN FLUID FLOW

Dr. Andrej Zlatos
Wisconsin University

Abstract

We consider an advection–diffusion equation on a compact Riemannian manifold. Our interest is in the question of enhancement of diffusion by a strong incompressible flow. Namely, we ask which flows make the deviation of the solution from its average arbitrarily small in arbitrarily short time, provided their amplitude is large enough. We give a sharp characterization of such “relaxation enhancing” flows in terms of the spectral properties of the dynamical system corresponding to the flow. This class includes, for instance, all weakly mixing flows. The result is proved by spectral methods in (and naturally extends to) a more general setting involving an anti–selfadjoint operator in place of advection and a positive self–adjoint operator in place of diffusion. In particular, we employ the RAGE theorem describing the evolution of a quantum particle corresponding to the continuous spectral subspace of a hamiltonian.

15 de agosto de 2005.

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PROPIEDADES ESPECTRALES DE UN OPERADOR NO AUTO–ADJUNTO

Dra. Carmen Martínez Adame
Universidad Nacional Autónoma de México

Resumen

Obtenemos cotas para el espectro del operador de Anderson no auto–adjunto en una dimensión utilizando rangos numéricos de orden superiores y en una gran variedad de casos podemos determinar por completo al espectro de nuestro operador.

26 de mayo de 2005.

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MATRICES DE JACOBI EN LA FÍSICA MATEMÁTICA CONTEMPORÁNEA

Dr. Luis O. Silva
Universidad Nacional Autónoma de México

Resumen

Desde hace más de un siglo las matrices de Jacobi han jugado un papel importante en el desarrollo del análisis matemático. A partir de 1930 se reconoce también el lugar central de estas matrices en la teoría espectral de operadores. Sin embargo, sólo recientemente se empieza a entender la relación entre las entradas de una matriz de Jacobi y sus propiedades espectrales, lo que ha tenido repercusiones importantes en la física matemática contemporánea.

12 de mayo de 2005.

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BOUND STATES ESSENTIAL SPECTRA OF SCHRÖDINGER OPERATORS

Dr. David Damanik
California Institute of Technology

Abstract

We present recent results that uncover a surprising relation between the bound states of Schrödinger operators and the spectral type inside the essential spectrum.

19 de abril de 2005.

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SOBRE LA ESTRUCTURA DE CHOQUES TERMOVISCOSOS

Dr. Pablo Luis Rendón
Universidad Nacional Autónoma de México

Resumen

La ecuación de Burgers se utiliza a menudo como la ecuación más sencilla que exhibe los efectos no lineales de una redistribución de energía y los efectos difusivos de la viscosidad sobre una región localizada. Para propagación plana, esta ecuación tiene la curiosa propiedad de ser integrable. En el

marco de la teoría de choques débiles se proponen soluciones de esta ecuación con forma de onda viajera, conocidas como choques de Taylor. En esta plática se presenta un estudio del límite de baja difusividad de la ecuación plana de Burgers, donde se introducen fluctuaciones pequeñas detrás de un choque de Taylor estacionario. Se obtienen resultados interesantes sobre la estructura misma de este choque, en particular sobre posibles deformaciones acompañadas de un ensanchamiento de la región más delgada.

14 de abril de 2005.

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MATRICES DE JACOBI EN LA FÍSICA MATEMÁTICA CONTEMPORÁNEA

Dr. Luis O. Silva
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Resumen

Desde hace más de un siglo las matrices de Jacobi han jugado un papel importante en el desarrollo del análisis matemático. A partir de 1930 se reconoce también el lugar central de estas matrices en la teoría espectral de operadores. Sin embargo sólo recientemente se empieza a entender la relación entre las entradas de una matriz de Jacobi y sus propiedades espectrales, lo que ha tenido repercusiones importantes en la física matemática contemporánea.

12 de mayo de 2005.

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PERTURBATIVE RESULTS FOR PRODUCTS OF RANDOM MATRICES

Dr. Hermann Schulz–Baldes
Erlangen University

Abstract

Random products of symplectic matrices naturally appear in the study of quasi one–dimensional random Schrödinger operators. In many physically interesting situations, there is a coupling constant controlling the degree of non–commutativity. Perturbative techniques then allow to calculate the associate persistant quantities, namely the Lyapunov exponents, the variance in the associated central limit theorem and moments of the invariant measure. Applications concern the Anderson model on a strip, a test of single parameter scaling in one dimension as well as the random polymer model.

3 de marzo de 2005.

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TEOREMA DE BORG–MARCHENKO PARA OPERADORES CON ESPECTRO CONTINUO

Dr. Ricardo Weder
Universidad Nacional Autónoma de México

24 de febrero de 2005.