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Vortex Motion on Bidimensional Fluids

The study of the dynamics of point-vortices in the plane, or other two-dimensional manifolds, is currently a very active field which involves Hamiltonian methods and techniques, some of which are borrowed from celestial mechanics; a good introduction to this subject is P. K. Newton, *The N-Vortex Problem: Analytical Techniques*, Vol. 145, Appl. Math. Sciences, Springer Verlag, New York, 2001.

The goal of this course is to describe the Hamiltonian equations of motion for the dynamics of N-point vortices in planar incompressible fluids. This problem presents many similarities as well as differences with the N-body problem of celestial mechanics.

We begin with Euler equations for the dynamics of incompressible fluids, showing how the logarithmic Hamiltonian for two dimensional fluids arises, exhibiting the first integrals and an invariant of the motion. Then we study the symmetries of the motion. After describing a few general properties for N-vortices, we specialize to problems of 3- and 4-vortices, studying mainly integrability, relative equilibrium solutions, boundedness of motion, and the possibility of collisions of vortices.

Contents:

1. Euler equations for incompresible fluids.
2. Planar N-vortex motion, Hamiltonian equations of motion.
3. First integrals and invariant of the motion. Symmetries. Relative equilibrium solutions.
4. Problems of 2- and 3-vortices as integrable Hamiltonian systems. Any total collapse solution of 3-vortices must be self-similar. Binary collisions are ruled out for problems of 2- and 3-vortices.
5. Problems of 4-vortices are in general non integrable. However, if the linear momentum and total vorticity are null, the problem is integrable. The same is true for 4-vortices in parallelogram configurations.