

Some Aspects of Random Schrödinger Operators and Random Spin Hamiltonians

Systems of condensed-matter physics with macroscopic disorder are modelled by Hamiltonians with parameters, which are randomly distributed according to some probability measure.

For example, one-particle Schrödinger operators with random potentials are used to caricature the phenomenon of Anderson localization, that is, the suppression of the tunnelling effect which yields in perfect solids, in other words ordered crystals, a purely absolutely continuous energy spectrum and macroscopic charge transport by the (Bloch-Floquet)electrons.

Another example is given by Hamiltonians for macroscopically many spins interacting with each other through random coupling constants and/or with a random external magnetic field. Here one wants to better understand the static and dynamic properties of spin glasses which at low temperatures exhibit a characteristic phase without long-range order because of “magnetic frustration”.

My course will have two parts of different lengths. The first (and longer) one will be devoted to random Schrödinger operators, the second one to spin glasses. Both parts will start with an introduction and then address more specific topics which are chosen mainly according to my own (research) experience.

In the first part I will concentrate on continuum models for which I will present and explain results about the integrated density-of-states, about the nature of the almost-sure energy spectrum and about anisotropic transport in a random magnetic field. In the second part I will basically restrict myself to (classical and quantum) mean-field type models in the sense of Sherrington/Kirkpatrick and discuss the almost-sure existence of the corresponding free energies.

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