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Central Configurations in Celestial Mechanics

The N -body problem describes the motion of N -point particles under the Newtonian attraction law. Celestial Mechanics is the branch of the mathematics which studies the N -body problem and all related aspects as for instance collisions, escapes, periodic orbits, singularities, bifurcations, etc.

In the 2-body problem, or Kepler problem or central force problem, we know exactly how are all motions. However very few is known for the N -body problem when $N \geq 3$.

A central configuration is a particular configuration of the N -particles where the acceleration vector of each particle is proportional to the corresponding position vector, and the constant of proportionality is the same for the N -particles. Many and important are the properties of the central configurations; for example, for any planar central configurations there are initial conditions which give rise to periodic orbits, where each particle is in uniform circular motion about the center of mass, and the configuration is maintained for all time. The system formed by the Sun, Jupiter and a Trojan asteroid is a well known example for the equilateral triangle central configuration application.

In this course we start studying the integrals of motion and the basic properties of the N -body problem, which allow us to have the frame work to study the central configurations. We will study the relationship between the Kepler problem and a central configuration, describing the geometric properties of the force vectors. We will give several examples pointing out the important contributions of L. Euler and J.L. Lagrange. The collinear central configurations were studied by Moulton, generalizing Euler's ideas. We will study this theorem and its applications. The central configurations give rise to the unique explicit solutions of the N -body problem known until now, these solutions are called homographic solutions, they play an important role in the understanding of the dynamics of particles. During this course we also will study these kind of solutions. We will finish the course by giving some open problems on the subject.