

From Random Schrödinger Operators to Random Matrices

Random Schrödinger operators have been a very active field of mathematical and physical research in the last 30 years, with contributions coming from all over the world. The basic question concerns the dynamics of quantum mechanical wave packets in a random environment (describing typically electrons in a doped semi-conductor). While the so-called strong Anderson localization regime is by now rather well under control, the diffusive weak-localization regime is at least mathematically still not well understood. In particular, the appearance of random matrix theory still remains unclear, even though its relevance is clearly evident from numerical results. The course aims to introduce the students to this field. A more specific outline is:

- Introduction to both topics in the title.
- Statement of main open problems. Two lines of attack are discussed in detail.
 - First approach: one performs a formal projection of the random Hamiltonian on the Fermi surface and then studying the obtained random matrix model (approach suggested by several other authors, but conclusive study of the toy model only in [SS1,SS2]).
 - Second approach: this is based on a detailed study of the random symplectic transfer matrices in a regime of weak coupling. The transfer matrix methods are first extended in various ways (matrix-valued Sturm-Liouville and Weyl limit point/limit circle theory), then a rigorous perturbation theory is developed based on the use of Fokker-Planck operators on the Lagrangian Grassmannians. Most of these are results of research carried out by Schulz-Baldes, Römer and Sadel in recent years [SB1,RS,SB2,SS3], and the techniques might be of broader use.

[SS1] J. Schenker, H. Schulz-Baldes, *Semicircle law and freeness for random matrices with symmetries or correlations*, Mathematical Research Letters 12, 531-542 (2005)

- [SS2] J. Schenker, H. Schulz-Baldes, *Gaussian fluctuations for random matrices with correlated entries*, IRMN, (2007).
- [SB1] H. Schulz-Baldes, *Perturbation theory for an Anderson model on a strip*, GAFA **14**, 1089-1117 (2004).
- [RS] R. Römer, H. Schulz-Baldes, *Weak disorder expansion for localization lengths of quasi-1D systems*, Euro. Phys. Lett. **68**, 247-253 (2004).
- [SB2] H. Schulz-Baldes, *Rotation numbers of Jacobi matrices with matrix entries*, preprint 2006.
- [SS3] C. Sadel, H. Schulz-Baldes, *Random Lie group actions on compact manifolds in a perturbative regime*, preprint 2007.