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Inverse Scattering in Quantum Mechanics

The inverse scattering problems in quantum mechanics have been extensively studied since the pioneering work of W. Heisenberg in the theory of the scattering matrix in 1943 and 1944. In these lectures we will discuss the multi-dimensional case, where we do not assume that the potentials are spherically symmetric. In general terms, they can be described as the problems of obtaining information about the inter-particle interaction potentials of an N-body system from the scattering operator. In a more precise sense, there are three main problems. Uniqueness: to prove that the scattering operator uniquely determines the potentials. Reconstruction: to give methods to reconstruct the potentials from the scattering operator. Characterization: to give necessary and sufficient conditions in order that an operator is the scattering operator of unique inter-particle potentials in a given class. There are different ways to give the scattering data. For example, one can give the scattering operator for all energies, the high-energy limit of the scattering operator, or the scattering operator for a fixed energy. Inverse scattering has many important applications in physics. In particular, in atomic molecular and nuclear physics a great deal of the information about the inter-particle potentials is obtained from scattering experiments. Moreover, there is also the closely related problem of inverse scattering of acoustic, electromagnetic and elastic waves, that has many technological applications, for example in tomography. Many of the results have been obtained using stationary methods. For general references on this point of view, see [1, 6, 7, 8, 9].

In these lectures I will discuss a new geometrical time-dependent method that has been introduced in [2, 3, 4]. We consider scattering of finite-energy wave packets, what allows us to use in the proofs the time propagation aspects of scattering. This makes the physical interpretation of the inversion methods transparent, and it is furthermore technically useful, for example, for N-body systems. Moreover, it allows us to consider more general classes of potentials with singularities.

The lectures will be organized as follows. We will first introduce the method in the simple case of two-body scattering by a short-range potential. Then, we will consider the case of N-body systems with long-range potentials. We will also briefly discuss the application of the method to other inverse scattering problems, like scattering by magnetic fields, the Aharonov-Bohm effect, the Stark effect, and the non-linear Schroedinger and Klein-Gordon equations. References for the material that we will discuss are [2, 3, 4, 5, 10, 11, 12, 13, 14, 15, 16, 17, 18].

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