International Workshop on PDEs and Material Sciences

IIMAS, UNAM, Mexico City

Tuesday, October 21st, 2025
Room B-204

Organizers:

- Luis Fernando López-Ríos (IIMAS, UNAM)
- Ramón G. Plaza (IIMAS, UNAM)

Invited Speakers:

- **Enrique Álvarez** (IIMAS, UNAM)
- **Felipe Angeles** (IIMAS, UNAM)
- **Davide Cusseddu** (Politecnico di Torino, Italy)
- **César Hernández-Melo** (State University of Maringá, Brazil)
- **Lauro Morales** (UAM, Iztapalapa)
- **Fabio Vallejo** (IIMAS, UNAM)

Sponsors:

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Program

	Tuesday, October 21st. Room B-204, IIMAS
10:00 - 10:40	**Felipe Angeles**: *Well-posedness of the compressible Navier-Stokes equations with fractional dissipation.*
10:50 - 11:30	**César Hernández-Melo**: *Lie Theory and instability of equilibrium solutions for logarithmic heat equations in Banach algebras.*
11:30 - 12:00	Coffee break
12:00 - 12:40	**Davide Cusseddu**: *Modelling spatial dynamics in Biology: cell polarisation and ecological patterning.*
12:40 - 15:00	Lunch break
15:00 - 15:40	**Lauro Morales**: *Generalized convex hull for the coplanar n-well problem and its relation to pattern formation in thin-film shape memory alloys.*
15:50 - 16:30	**Enrique Álvarez**: *Existence and spectral stability analysis of a family of standing periodic wave solutions for a modified Eckhaus equation with an additional term.*
16:30 - 17:00	Coffee break
17:00 - 17:40	**Fabio Vallejo**: *Nonlinear dynamic stability of phase transitions in hyperelastic materials.*

Abstracts:

- **Felipe Angeles**: *Well-posedness of the compressible Navier-Stokes equations with fractional dissipation.*

Abstract: We study the Cauchy problem for the compressible Navier-Stokes equations with viscosity given by the fractional Laplacian. In this talk, we study the local existence and uniqueness of solutions for this system of equations in hypo-dissipative regimes. Then, under the assumption of an initial condition sufficiently close to a constant equilibrium state, we show the global existence of solutions. In order to compute the time decay rates of the solution to the equilibrium state, we introduce the concept of fractional compensating matrix. In particular, we generalize Kawashima-type estimates known in the theory of hyperbolic-parabolic systems of equations to the case of symmetric-hyperbolic systems with a fractional regularization.

- **César Hernández-Melo**: *Lie Theory and instability of equilibrium solutions for logarithmic heat equations in Banach algebras.*

Abstract: Let $\mathbb{X}$ be a commutative Banach algebra with identity denoted by e. This work investigates the existence and stability of equilibrium solutions for the nonlinear reaction-diffusion equation:

$$u_t = \Delta u + wu + \mathrm{Ln}(u^2)u,$$

where u is a function from $\mathbb{R}^n \times \mathbb{R}^+$ to $\mathbb{X}$, $w \in \mathbb{X}$ is a fixed parameter and Ln denotes the logarithm function defined on the open ball $B(e,1)$. We derive an explicit formula, using exponential functions, for a family of equilibrium solutions $\phi_r : \mathbb{R}^n \to \mathbb{X}$ that decay to zero at infinity. The instability of these equilibria is established through two methods: 1) At the linear level, we analyze the spectral properties of the linear operator $DF(\phi_r)$ where F denotes the nonlinear field $F(u) = \Delta u + wu + \mathrm{Ln}(u^2)u$; the analysis of the spectrum relies on the fact that the linear operator $DF(\phi_r)$ is part of a specific Lie-algebra. 2) At the non-linear level, we analyze the behavior of particular curves of solutions that are perturbations of the equilibrium solutions. This approach is also used to demonstrate the nonlinear instability of any nontrivial equilibrium solution of the equation. Finally, we show that the nonlinear instability implies the linear instability by utilizing a specific Lie symmetry of the equation.

- **Davide Cusseddu**: *Modelling spatial dynamics in Biology: cell polarisation and ecological patterning.*

Abstract: In the first part of this seminar, I will present the bulk–surface wave–pinning model, which constitutes one of the simplest models of cell polarisation. It is a toy model that is derived from minimal key properties of GTPase proteins. In particular, the model takes into account the activation/inactivation dynamics as well as the spatial cycling of proteins between the cell membrane and the cytosol. The bulk–surface framework allows for the description of cytosolic-cell membrane interactions through boundary conditions. After presenting the model, we will discuss some results and numerical simulations on different three-dimensional geometries. In the second part, I will introduce a more recent work on modelling ecological dynamics. Here we consider two structured heterogeneous populations, subject to fast phenotypic mutations. Under certain conditions, we are able to recover classical ecological models, in form of reaction–diffusion equations, giving rise to spatial segregation and patterning.

- **Lauro Morales**: *Generalized convex hull for the coplanar n-well problem and its relation to pattern formation in thin-film shape memory alloys.*

Abstract: In shape-memory alloys, it is common to analyze pattern formation induced by the existence of a finite number of zero-energy material phases $U \subset \mathbb{R}^{n\times n}$. These phases are characterized as minima of a non-convex bulk energy functional, $I(U)$, which exhibits scale-invariance. This invariance leads to the existence of minimizing sequences which weakly converge to some "average" phases that are not minima of $I(U)$ and generate the observed microstructure in the sample. The set of all constant weak limits is known as the *quasiconvex hull*, QU. Determining QU is a challenging task, and only a few examples of sets U have explicit sets QU. In this talk, we investigate the set QU for a finite set $U \subset \mathbb{R}^{2\times 2}_{\text{sym}}$. We will identify an explicit set BU, contained in the convex hull CU of the wells, such that $QU \subseteq BU \subset CU$. Additionally, we will explore the conditions on the well distribution to attain the equality between QU and BU.

- **Enrique Álvarez**: *Existence and spectral stability analysis of a family of standing periodic wave solutions for a modified Eckhaus equation with an additional term.*

Abstract: We study the existence and spectral stability properties of a family of periodic standing wave solutions to a modified version of Eckhaus equation that incorporates an additional term. The family consists of small amplitude waves with finite fundamental period which emerge from a Hopf bifurcation around a critical value of a specific parameter. It is shown that the Floquet (continuous) spectrum of the linearization around the periodic standing waves intersects the unstable half plane of complex values with positive real part. To conclude this, we decompose the linearized operator as the sum of a constant coefficient operator, followed by a first order perturbation and then a second order perturbation. We prove that the small-amplitude stationary periodic wave solutions are spectrally unstable by applying techniques from perturbation theory of linear operators.

- **Fabio Vallejo**: *Nonlinear dynamic stability of phase transitions in hyperelastic materials.*

Abstract: Within the framework of nonlinear elasticity, it is known that a hyperelastic material with a nonconvex energy density function (with multiple wells) subjected to loading can reach a minimum energy state by forming a two-phase microstructure, where two deformation states coexist and meet along a surface or interface. This configuration, known as a phase transition, can be understood as an average deformation that minimizes the total energy. This talk addresses phase transitions in hyperelastic materials governed by regular kinetic relations that describe the driving force along the interface. It will be shown that the nonlinear dynamic stability problem of such configurations reduces to the verification of algebraic conditions known as Kreiss–Lopatinskii conditions, and some stability results for specific cases will be presented.